Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Family name:	**Department:**
First name:	**ETH ID No.:**

For the grading:

	1K	**2K**	**Points**	**Comments:**
1				
2				
3				
4				
5-12				
Total				

MATHEMATICS I EXAM

for students of Agricultural Science, Earth Sciences, Environmental Sciences, and Food Science

Important:

- Please fill the header on the cover page and lay your ETH-card visible on the table.
- Please write neatly with a non erasable blue or black pen, in particular not with a pencil. Beware that something that is too hard to read could be ignored.
- Please leave some empty space on the margins for the correction.
- This exam has 12 questions and lasts for 90 minutes.

For questions 1-4:

- Please write down all intermediate steps of your calculations and solutions.
- Write your name and ETH ID / Legi-Nr. on each additional sheet.
- The maximal score of each exercise part is given in the right margin.

For questions 5-12:

- Mark your answers clearly.
- There is always only one correct answer and 2 points per question.

Permitted aids:

- Written notes up to 20 A4-Pages, one English dictionary,
- **no** calculator, **no** mobile phone, **no** laptop.
- Please switch off your mobile phone and stow it away.

Good Luck!

1. Consider the function
$$f(x) = \frac{1}{1 - \tan(x)}.$$

a) Determine the derivative $f'(x)$. 2 points

b) Determine the linearization of $f(x)$ in $x_0 = 0$. 2 points

c) Determine the range of $\tan(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{4}$. 2 points

d) Determine the range of $f(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{4}$. 3 points

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2. Determine the general solution of the the following differential equations:

a) $y'' = 4y' - 4y$. 4 points

b) $3xy' - y = x + 1$ for $x > 0$. 4 points

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3. Consider the matrix
$$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 4 \\ 3 & 6 & 3 & 9 \end{pmatrix}.$$

a) Determine the rank of the matrix A. 3 points

b) Determine a basis for the solution set of the matrix equation $A\vec{x} = \vec{0}$. 3 points

c) Let $\vec{b}$ be the sum of all four columns of A. Determine the general solution of the system $A\vec{x} = \vec{b}$. 2 points

4. Consider the following system of differential equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 \\ 4 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix}.$$

a) Determine the eigenvalues and the corresponding eigenvectors of the coefficient matrix A of the system. 3 points

b) Determine the solution of the system with the initial value

$$\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

3 points

c) Find all values k such that every solution of the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ k & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

is bounded for all $t \in \mathbb{R}$. 3 points

For questions 5-12: Each question gives 2 points. Wrong or multiple answers give 0 points. Mark your answers on this exam.

5. What is the limit

$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2} \ ?$$

(a) $-\infty$.

(b) $\frac{1}{2}$.

(c) 0.

(d) $+\infty$.

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6. On which interval does the function

$$f(x) = (x-1)^3(4-x)^2$$

attain an extremum?

(a) $]0, 1[$.

(b) $]1, 2[$.

(c) $]2, 3[$.

(d) $]3, 4[$.

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7. The differential equation

$$f' = (f+2)(f+3)$$

is

(a) non-linear of order 1.

(b) linear homogeneous of order 1.

(c) non-linear of order 2.

(d) linear homogeneous of order 2.

8. A curve with measured data has the form $b = \frac{2}{3}a$ in a log-log plot (i.e. instead of x and y we use $a = \log_{10} x$ and $b = \log_{10} y$ on both, the horizontal and vertical axes). Which function $y = f(x)$ does the curve represent?

(a) $y = x^{\frac{2}{3}}$.

(b) $y = x^{\frac{3}{2}}$.

(c) $y = 10^{\frac{3x}{2}}$.

(d) $y = 10^{\frac{2x}{3}}$.

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9. The expression

$$\frac{-i-7}{2+i}$$

is equal to

(a) $-3 - i$.

(b) $-3 + i$.

(c) $3 - i$.

(d) $3 + i$.

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10. The zeros of the polynomial

$$p(\lambda) = \lambda^3 + 8$$

are

(a) -2, $2i$, $-2i$.

(b) -2, $\sqrt{2} + \sqrt{2}i$, $\sqrt{2} - \sqrt{2}i$.

(c) -2, $2e^{i\frac{2\pi}{3}}$, $2e^{i\frac{4\pi}{3}}$.

(d) -2, $2e^{i\frac{\pi}{3}}$, $2e^{i\frac{5\pi}{3}}$.

11. The determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 3 & 4 & 1 & 5 & 2 \\ 4 & 5 & 0 & 2 & 3 \\ 5 & 6 & 0 & 0 & 1 \end{pmatrix}$$

is

(a) -3.

(b) -2.

(c) 2.

(d) 3.

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12. Let

$$A = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}.$$

Which of the following statements is **false**?

(a) The matrix A is diagonalizable.

(b) The matrix A has two distinct eigenvalues.

(c) The columns of A build a basis of $\mathbb{R}^2$.

(d) The columns of A form an eigenbasis for A.