Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Family name:	**Department:**
First name:	**ETH ID No.:**

For the grading:

	1K	2K	Points	Comments:
1				
2				
3				
4-13				
Total				

MATHEMATICS I EXAM

for students of Agricultural Science, Earth Sciences, Environmental Sciences, and Food Science

Important:

- Please fill the header on the cover page and lay your ETH-card visible on the table.
- Please write neatly with a non erasable blue or black pen, in particular not with a pencil. Beware that something that is too hard to read could be ignored.
- Please leave some empty space on the margins for the correction.
- This exam has 13 questions and lasts for 90 minutes.

For questions 1-3:

- Please write down all intermediate steps of your calculations and solutions.
- Write your name and ETH ID / Legi-Nr. on each additional sheet.
- The maximal score of each exercise part is given in the right margin.

For questions 4-13:

- Mark your answers clearly.
- There is always only one correct answer and 2 points per question.

Permitted aids:

- Written notes up to 20 A4-Pages, one English dictionary,
- **no** calculator, **no** mobile phone, **no** laptop.
- Please switch off your mobile phone and stow it away.

Good Luck!

1. Consider the function

$$f(x) = \frac{e^{3x}}{x} \text{ for } x \textbf{ positive}.$$

a) Determine and classify the local extrema of $f(x)$. 4 points

b) Determine the range of $f(x)$. 3 points

c) Let $F(x),\ x > 0$ be a function with

$$\begin{cases} F'(x) = f(x) \\ F(1) = 0 \end{cases}$$

and let $G(x)$ be the inverse function of $F(x)$. Then we have that $G(0) = 1$. Determine $G'(0)$. You **do not** have to determine $F(x)$. 3 points

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2. Determine the general solution of each of the following differential equations:

a) $y'' + 2\sqrt{2}y' + 2 = 0$ 5 points

b) $y' - 2xy - x = 0$ für $x > 0$. 5 points

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3. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 4 & 1 \end{pmatrix}.$$

a) Determine the eigenvalues of A. 4 points

b) Is A diagonalizable? 2 points

c) For which vectors $\vec{x}_0$ is

$$\vec{x}(t) = e^t \vec{x}_0$$

a solution of

$$\dot{\vec{x}} = A\vec{x} \quad ?$$

4 points

For exercises 4-13: Each question gives 2 points. Wrong or multiple answers give 0 points. Mark your answers on these exam sheets by circling the right answer.

4. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 2 \end{pmatrix}.$$

What are the rank and the dimension of the kernel of A?

(a) $\operatorname{rank}(A) = 2$ und $\dim(\ker(A)) = 0$.

(b) $\operatorname{rank}(A) = 2$ und $\dim(\ker(A)) = 1$.

(c) $\operatorname{rank}(A) = 3$ und $\dim(\ker(A)) = 0$.

(d) $\operatorname{rank}(A) = 3$ und $\dim(\ker(A)) = 1$.

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5. Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ -4 & 5 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 5 & 0 \\ 0 & 3 & -4 \\ 0 & -1 & 2 \end{pmatrix}.$$

Which of the following claims is **wrong**?

(a) $\det(2B^{-1}A^{-1}) = -1$.

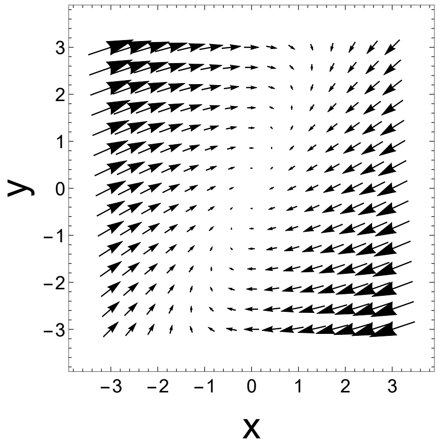
(b) $\det(-B^{-1}A^2) = 1$.

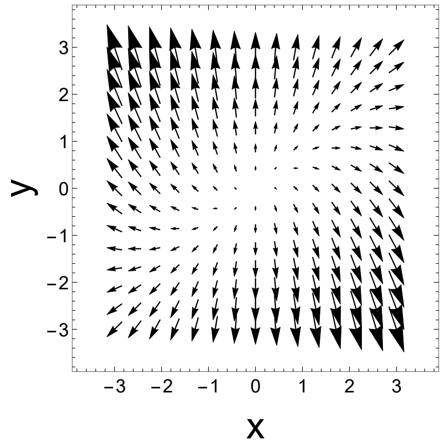
(c) $\det(2AB^{-1}) = -4$.

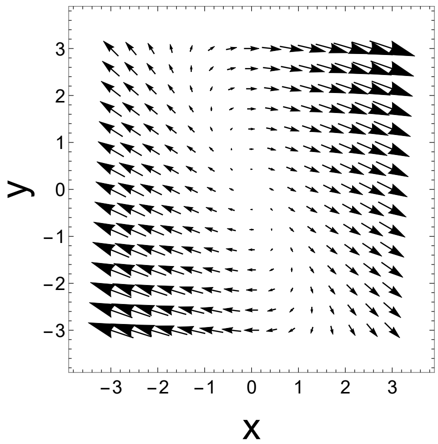
(d) $\det(-2A^{-1}) = 4$.

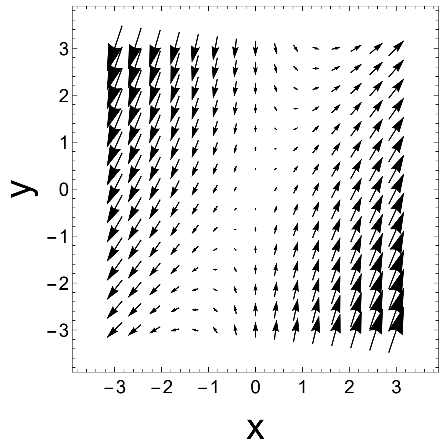
6. Which picture shows the phase portrait of the system

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \vec{x} \quad ?$$

(a)


(c)


(b)


(d)


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7. Which of the following limits exist?

(I) $\lim_{x\to 0} \frac{e^{2x}-2e^{x}+1}{x^2}$

(II) $\lim_{x\to +\infty} \frac{\cos(x)}{\ln(x)}$

(a) Both limits exist.

(b) Limit (I) exists, but limit (II) does not exist.

(c) Limit (I) does not exist, but limit (II) exists.

(d) Both limits do not exist.

8. The expression $\frac{(2-4i)^2}{i-3}$ can be transformed into

(a) $-2-6i$.

(b) $-2+6i$.

(c) $2-6i$.

(d) $2+6i$.

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9. Let $z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$. What is the real part of z^9?

(a) $-\frac{\sqrt{3}}{2}$

(b) -1

(c) 0

(d) $\frac{1}{2}$

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10. What is the derivative of the function

$$f(x) = \int_{e^{-x}}^{0} \cos(t^2)\, dt$$

at the point $x = 0$?

(a) -1.

(b) $-\cos(1)$.

(c) $\cos(1)$.

(d) 1.

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11. Which is the general solution of the following differential equation:

$$2y'' + 2y = 2x + 1 \text{ ?}$$

(a) $k_1 \cos(x) + k_2 \sin(x) + 2x + 1$

(b) $k_1 \cos(x) + k_2 \sin(x) + x + \frac{1}{2}$

(c) $k_1 \cos(2x) + k_2 \sin(2x) + x + \frac{1}{2}$

(d) $k_1 \cos(2x) + k_2 \sin(2x) + 2x + 1$

12. The integral

$$\int_{-\pi}^{\pi} x \cdot \cos^2(x)\, dx \ ?$$

is equal to

(a) 0

(b) 2π

(c) π

(d) 1

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13. For which value k is the function

$$f(x) = \begin{cases} e^{k^3 x} & \text{for } x \geq 0, \\ \sin(8x) + 1 & \text{for } x < 0, \end{cases}$$

differentiable at every point in $\mathbb{R}$?

(a) -2

(b) k can be arbitrary

(c) There is no such value k.

(d) 2
