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Editors

***RIGIDITY IN DYNAMICS
AND GEOMETRY***

*Contributions from the Programme “Ergodic Theory,
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Preface

This volume of proceedings is an offspring of the special semester *Ergodic Theory, Geometric Rigidity and Number Theory* which was held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK, from January until July, 2000. Beside the activities during the semester, there were workshops held in January, March and July, the first being of introductory nature with five short courses delivered over a week. Although the quality of the workshops was excellent throughout the semester, the idea of these proceedings came about during the March workshop, which is hence more prominently represented,

The format of the volume has undergone many changes, but what has remained untouched is the enthusiasm of the contributors since the onset of the project: suffice it to say that even though only two months elapsed between the time we contacted the potential authors and the deadline to submit the papers, the deadline was respected in the vast majority of the cases.

The scope of the papers is not completely uniform throughout the volume, although there are some points in common. We asked the authors to write papers keeping in mind the idea that they should be accessible to students. At the same time, we wanted the papers not to be a summary of results that appeared somewhere else. There are several ways in which each author satisfied these requests: some papers are mostly surveys of a particular subject, some illustrate particularly enlightening cases of results whose technical proofs appear somewhere else, some provide new proofs of known results, yet some are completely original research papers. What is uniform throughout the volume is the quality of the writing, each contribution in its own way.

Each paper had at least two referees and was proofread an innumerable number of times, both by the authors and by the editors. We, the editors, apologize to the authors if in the attempt of making each paper “perfect”, we sometimes overdid it, and to the readers for not succeeding.

Finally we want to thank Dr. Joachim Heinze at Springer-Verlag, for his unconditional support and for accommodating all our requests. Our thanks go also to Ute Motz and Leonie Kunz from Springer-Verlag for their help and technical support in the preparation of this volume. Last but not least, thanks to the authors for making this volume possible.

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