

WELCOME EVERYBODY! ALLE HERZLICH WILLKOMMEN!

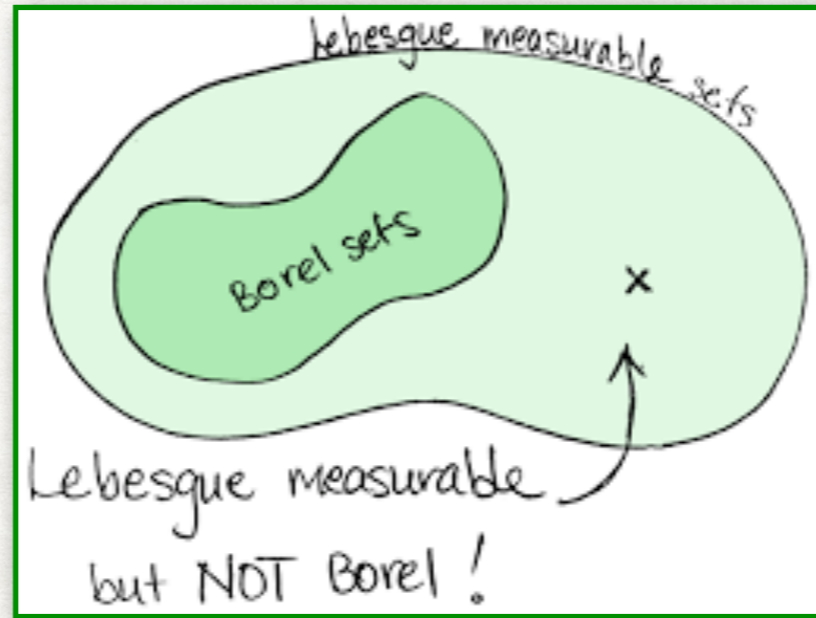
RIEMANN VS LEBESGUE INTEGRATION



$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$



The Lebesgue integral extends the scope of the Riemann integral. It was developed by Henri Lebesgue, who was born today, June 28, 1875



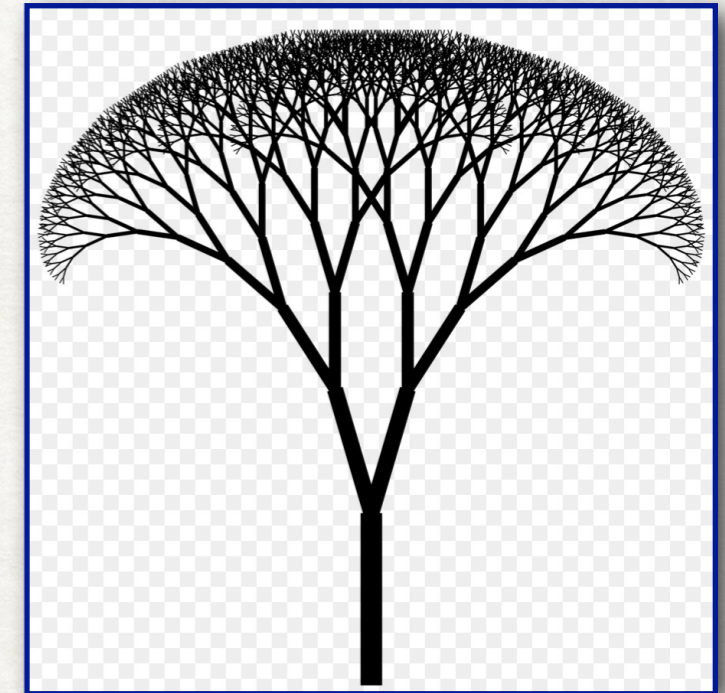
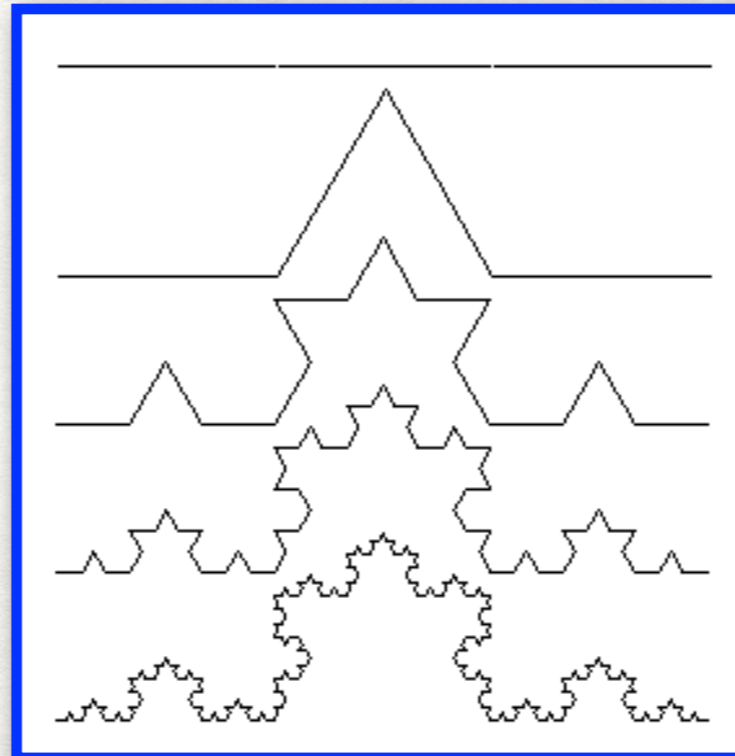
Measure Theory

Part 2

$$B(X) := \sigma(T) = \bigcap_{A \in T} \mathcal{A}$$

ENG

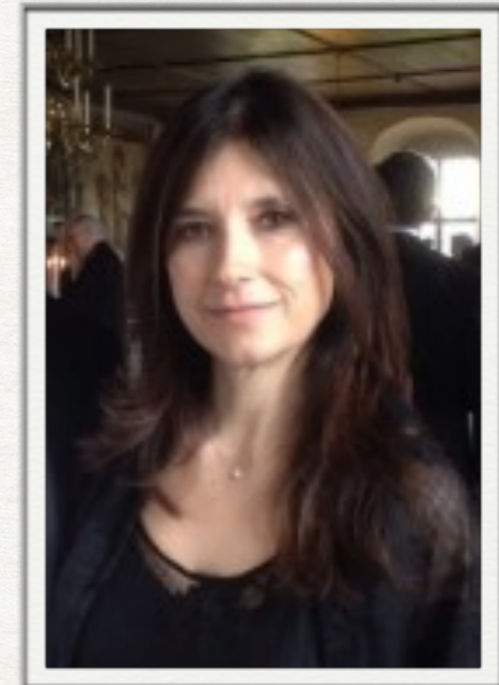
Borel
sigma-algebra



MASS UND INTEGRAL, D-MATH

401-2284-00L, SS2021

Lecturer: **Francesca Da Lio**
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Teaching Assistants



Salome Schumacher



Michael Vogel



Leon Staresinic



Riccardo Plati



Maran Mohanarangan

ADMINISTRATIVE INFORMATION

Course Webpage

<https://metaphor.ethz.ch/x/2021/fs/401-2284-00L/>

My Webpage (for Lecture Notes, Class Content and other Material)

<https://people.math.ethz.ch/~fdalio/MASSundINTEGRALFS21>

AND

<http://www.vorlesungsverzeichnis.ethz.ch>

INFORMATION LECTURES AND EXERCISES

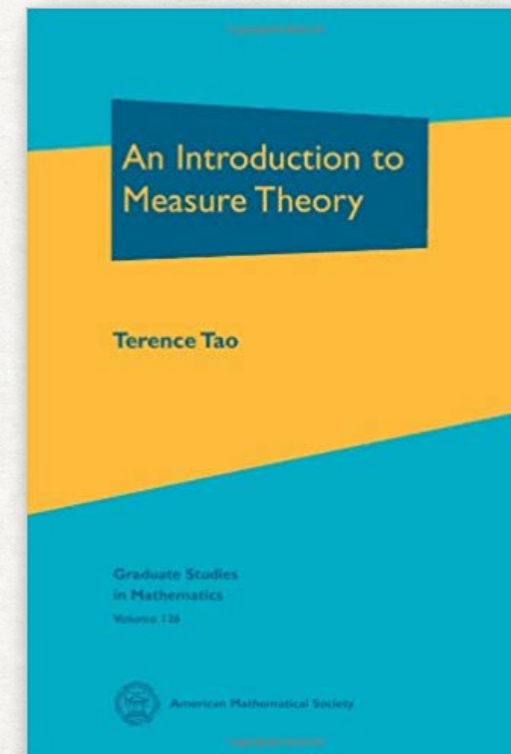
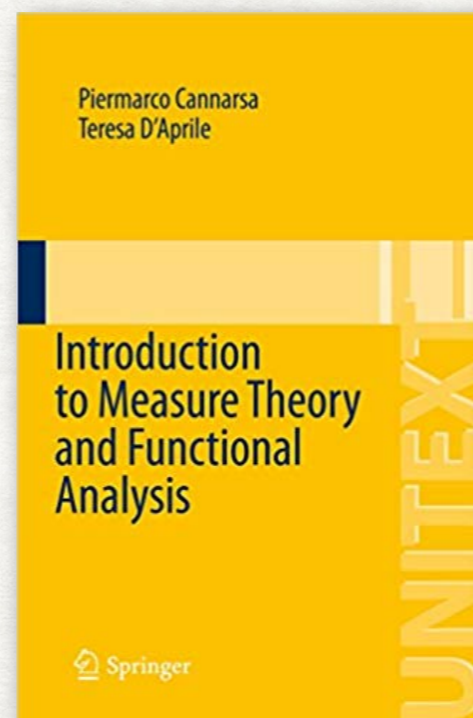
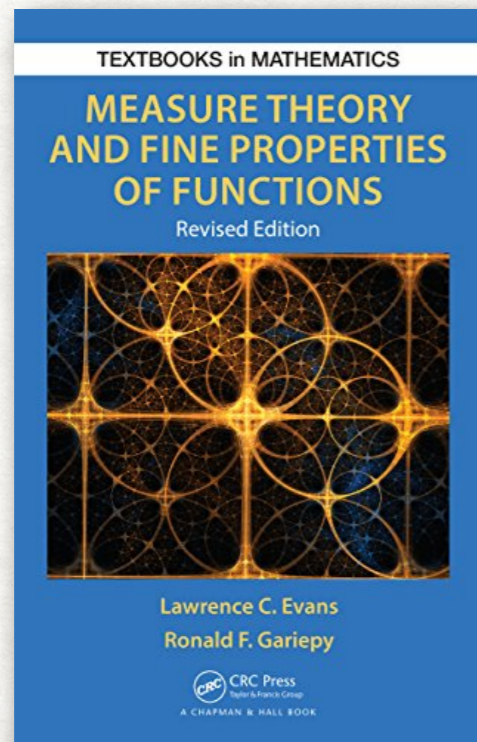
- Until at least April 1st the lectures will be on Zoom (the link to the recording will be posted on webpage of the course, a password will be sent you apart).
- Exercise classes in Zoom: in **English**: *Maran Mohanarangan, Riccardo Plati, Leon Staresinic*; in **German**: *Salome Schumacher, Michael Vogel*.
- Forum: There is a forum for the students to discuss the current exercise sheet and topics from the lecture. You can access the forum <https://forum.math.ethz.ch/>.
- You can also contact me by email: fdalio@math.ethz.ch

FURTHER INFORMATIONS

- **Evaluation:** *There will be a **20 minute oral exam (18 minutes exam, 3 minutes discussion of the grade)**: it will consist in **two questions** where you will have to prove two results (sometime if I am not satisfied or I want to be sure for the maximal grade I ask a 3rd question).*
- **Weakly homeworks:** *I really encourage active and regular participation to our weekly problem sessions: they will give you the opportunity to review the topics in smaller groups, to discuss problems and see some of them solved in great detail. I advise you to work in a timely manner. Studying Mathematics is effective if it is a regular activity. I advise you to attend as much as possible the lectures: they aim at guiding you in understanding the key concepts in each chapter*
- *During the lectures you can ask me questions either during the break or in the zoom chat. I will try to answer them either directly or during the break or by emails (it depends on the questions).*

TEXTBOOKS

- My **Lecture Notes** (in **English**) (which will be continuously updated. **Remark and comment are always welcome!**).
- **Michael Struwe's Notes: Analysis III, Mass und Integral** (in **German**)
- **Additional recommended bibliography:**



THIS COURSE:

The goal of this course is to provide notions of abstract measure and integral which are more general and robust than the notion of **Jordan measure** and **Riemann integral** (*for a nice presentation of Jordan measure and Riemann integral look for instance at the notes of Analysis 1 & 2 by Michael Struwe or the book by Terence Tao*).

Why do we need a finer concept of measure than the one we already have with the Jordan's measure?

1. From the **point of view of geometry**, we may be interested in being able to “measure” as many quantities as possible in a natural way. For this we need a measure with which we can also measure countable unions of measurable quantities. The Jordan measure cannot do this, as some examples show.
2. From the **point of view of the analysis** we need a theory of integration which extends Riemann's theory and concerns with a more general class of functions, not necessarily continuous or piecewise continuous (the so-called Borel or measurable functions).
3. Finally, abstract measure theory is also of fundamental importance for the **field of stochastics**, since calculating with probabilities is only possible in the language of measure theory.

PRELIMINARY PROGRAM

- Measure Spaces (Lebesgue Measure, Hausdorff Measure, Radon Measure)
- Measurable Functions: definition and properties
- Integration: definition, properties, theorems of convergence, Lebesgue L^p spaces
- Product Measures and Multiple Integrals. Fubini and Tonelli Theorems, Convolutions
- Differentiation of measures