# Welcome Everybody!
# Alle Herzlich Willkommen!

# Analysis III - D-MAVT, D-MATL
# 401-0363-10L, HS 2020

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# ADMINISTRATIVE INFORMATION

**Course Webpage:**

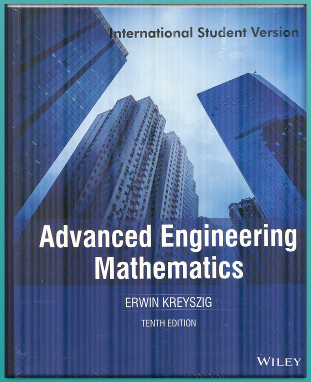
https://moodle-app2.let.ethz.ch/course/view.php?id=13495

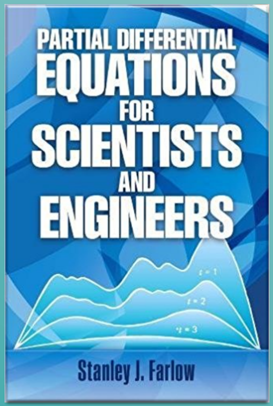
**and**

http://www.vorlesungsverzeichnis.ethz.ch

# Textbooks

- **Alessandra Iozzi's Lecture notes**
- **Class Notes**
- **Additional recommended bibliography:**



- **Lectures:** *Live streaming on Monday 14 - 16 at https://video.ethz.ch/live/lectures.html and recording in https://video.ethz.ch/lectures.html*

- **Exercises:** *Exercise classes are held every Thursday, starting from Thursday 24th September.*

- *The exercise sheet will be uploaded online every Monday on* ***Moodle*** *and it will be discussed with your TA on Thursday of the same week. The solutions will be uploaded online, after the deadline to hand-in your solutions (the Thursday after the discussion with the TA).*

- **EduApp :** *you can write each week questions about the lecture (course channel or clicker questions) and I will answer either during the break or the week later.*
  *Questions on exercises* → *Forum in Moodle*

- ***Evaluation:*** *There will be a 2 hour written final examination. More information will be provided towards the end of the semester.*

# CONTENT

The main goal is to study basic linear partial differential equations used widely in engineering applications.
We develop solution techniques through transformations and study their analytical properties:

- ***Laplace Transform***
- ***Fourier Analysis***
  - Fourier Series
  - Fourier Integrals

# What is a Partial Differential Equation (PDE)?

- *It relates state variables like mass, velocity, energy….to their variations with respect to space,time…..*
- *It provides an important modeling tool for the physical sciences, theoretical chemistry, biology, socio-economics sciences, engineering sciences….*
- *It governs many phenomena occurring in the nature around us*
- *PDE involve deep and beautiful mathematics*

# Recall from last year: evolution of the coffee temperature H(t)

H(t) satisfies the ODE:

$$\dot{H}(t) = H'(t) = \frac{dH}{dt}(t) = -\kappa(H(t) - H_{ext})$$

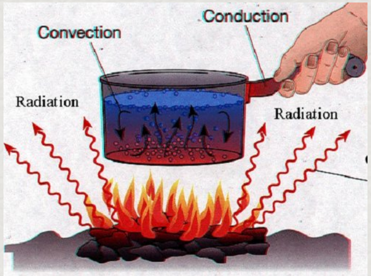
$H_{ext}$ is the external temperature. If $H(0) = H_0$ then

$$H(t) = (H_0 - H_{ext})e^{-kt} + H_{ext}$$

# Main Examples of PDEs

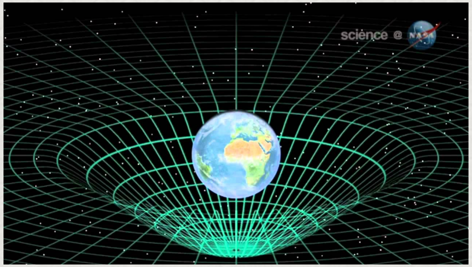
***Wave Equation***

$$u_{tt} - c^2 u_{xx} = 0$$




***Heat Equation***

$$u_t - k^2 u_{xx} = 0$$




***Potential/Laplace's Equation***

$$u_{tt} + u_{xx} = 0$$

- **Maxwell's Equation: description of electromagnetic phenomena**

- **Schrödinger's Equation: mathematical formulation for studying quantum mechanical systems:**

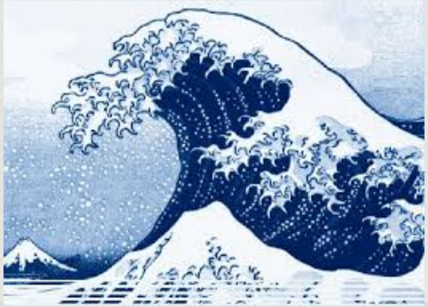
$$i\hbar u_t = -\frac{\hbar}{2m}\Delta u + Vu.$$

V is a known function (potential), m is the particle's mass and $\hbar$ is the Planck's constant

# OTHER EXAMPLES

- ***Navier-Stokes Equation***

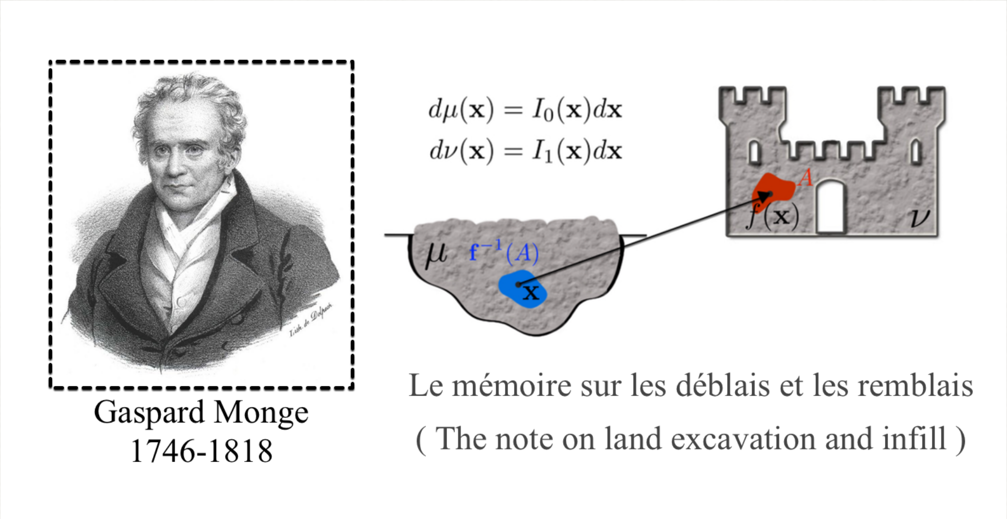
The Clay Mathematics Institute, a private American foundation, proposed a list of seven problems, the resolution of which will be rewarded with a prize of one million dollars for each. Most of these challenges are of great abstraction and may seem remote from any physical reality. By contrast, the problem of Navier-Stokes equations appears quite concrete in this list: these equations are supposed to describe the flow of ordinary fluids (liquids or gases) (http://images.math.cnrs.fr/Turbulences-sur-les-equations-des.html?lang=fr)



## Monge Ampère Equation

$$\mathrm{Det}\,(D^2 u) = f(x) \quad \text{in} \quad \Omega \subseteq R^n$$

**Application: Geometry, Optimal Transport…..**



# How do you solve a PDE?

1. Separation of Variables: *This technique reduces a PDE in n variables to a n ordinary differential equations (ODEs).*

2. Integral Transforms: *It reduces a PDE in n independent variables to one in n-1 variables.*

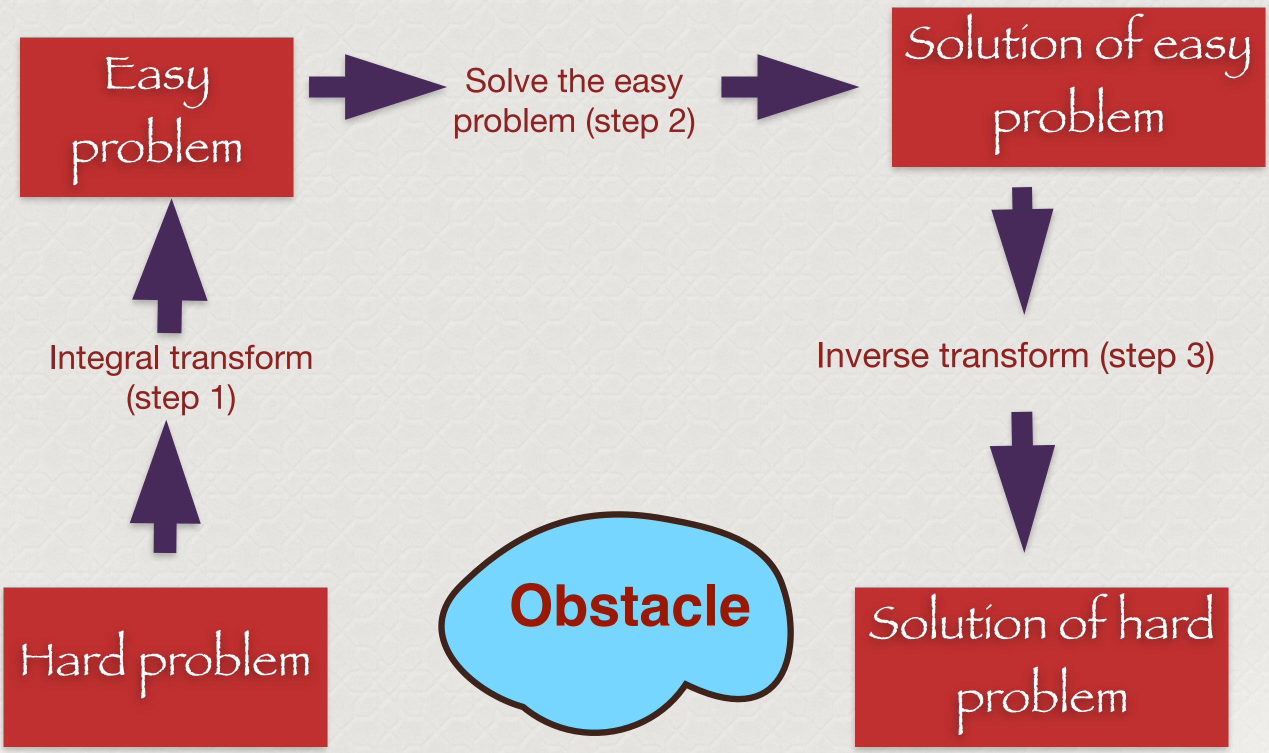
3. Change of Coordinates: *It changes the original PDE to an ODE or an easier PDE by changing the coordinates.*

4. Numerical Methods: *They change a PDE to a system of differential equations that can be solved by means of iterative techniques.*

5. …….

# Chapter 1 : The Laplace Transform

# Transform: Basic idea in solving mathematical problems



# The Laplace Transform uses this philosophy

solve the algebraic equation (easy)

$$sF(s) + F(s) = U(s) \longrightarrow F(s) = \frac{1}{s+1}U(s)$$

Laplace Transform

inverse Laplace Transform

$$\begin{cases} f'(t) + f(t) = u(t) \\ f(0) = 0 \end{cases} \longrightarrow f(t) = \int_0^t e^{t-\rho} u(\rho) d\rho$$

difficult

What is not revealed by this diagram is how to pass from the original equation

$$f'(t) + f(t) = u(t)$$

to the algebraic equation involving the Laplace Transform F(s) and then back from the solution F(s) of the algebraic equation to the solution f(t) of the original equation.

That's our next task: to develop the theory of Laplace Transform

***Thank you for your attention!***

# **Let's start!**