Program of Analysis III-Masstheorie HS23 ETH, D-MATH

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For the preparation of the **written and oral exam** you should take as guideline the class content that you find in

https://people.math.ethz.ch/~fdalio/Analysis3MeasureTheoryHS23

The written exam will consist of three exercises, one of which will include a set of **multiple choice questions** as in the bonus exercises of the series in HS23. Below you find the list of definitions and results (the enumeration is taken from the Lecture Notes of the course) that you should know for the oral and written exam. We advise you to look at also the class notes of the lectures (that you find in polybox).

CHAPTER 1: Remark 1.1.1, Definition 1.1.2, Remark 1.1.3, Remark 1.1.6, Definition 1.1.7, Definition 1.2.1, Remark 1.2.2, Definition 1.2.3, Exercise 1.2.4, Definition 1.2.5, Definition 1.2.7, Definition 1.2.9, Remark 1.2.10, Theorem 1.2.11 (proof), Definition 1.2.12, Theorem 1.2.14 (proof), Definition 1.2.16, Theorem 1.2.18 (proof), Theorem 1.2.20 (proof), Definition 1.3.1, Remark 1.3.3, Lemma 1.3.4 (proof), Definition 1.3.5, Definition 1.3.6, Theorem 1.3.7 (proof), Theorem 1.3.8 (proof), Corollary 1.3.9 (no proof), Theorem 1.4.1 (proof), Definition 1.4.3, Corollary 1.4.4 (no proof), Section 1.5: Vitali Set (construction and proof that it is not Lebesgue measurable), Exercise 1.5.1, Exercise 1.5.2, Section 1.6: Cantor Set (construction and proof it has Lebesgue measure zero and it is uncountable, Definition 1.7.1, Theorem 1.7.2

(no proof), Theorem 1.7.4 (no proof), Theorem 1.7.5 (no proof), Definition 1.8.1, Definition 1.8.2, Theorem 1.8.3 (proof), Remark 1.8.4 , Lemma 1.8.5 (proof), Example 1.8.6, Definition 1.8.8, Remark 1.8.9, Definition 1.9.1, Example 1.9.2.

CHAPTER 2: Definition 2.2.1, Remark 2.2.2, Remark 2.2.3, Theorem 2.2.5 (proof), Theorem 2.2.6 (proof), Remark 2.2.8, Theorem 2.3.1 (Egoroff) (proof), Exercise 2.3.2, Theorem 2.3.3 (Lusin) (no proof), Remarl 2.3.4, Definition 2.4.1, Theorem 2.4.2 (proof), Remark 2.4.3, Theorem 2.4.4 (proof).

CHAPTER 3: Definition 3.1.1 , Definition 3.1.2, Definition 3.1.3, Definition 3.1.4, Proposition 3.1.6 (proof), Proposition 3.1.7 (proof), Definition 3.1.8, Proposition 3.1.9 (proof), Proposition 3.1.10 (proof), Corollary 3.1.11 (proof), Theorem 3.1.12 (proof), Corollary 3.1.13 (proof), Theorem 3.1.14 (non proof), Corollary 3.1.15 (proof), Lemma 3.1.16 (no proof), Corollary 3.1.17 (no proof), Proposition 3.1.18 (no proof), Example 3.2.1, Proposition 3.2.2 (proof), Theorem 3.3.1 (Fatou's Lemma)(proof), Example 3.3.2, Theorem 3.3.3 (Monotone Convergence Theorem) (proof), Theorem 3.3.5 (Dominated Convergence Theorem (proof), Theorem 3.5.3 (proof), Definition 3.6.1, Theorem 3.6.2 (Vitali's Theorem)(proof), Remark 3.6.3. Definition 3.7.1, Remark 3.7.2, Remark 3.7.3, Theorem 3.7.5 (L^p spaces are Banach spaces) (proof), Lemma 3.7.6 (Young Inequality) (proof), Corollary 3.7.7 (Hölder Inequality) proof), Corollary 3.7.8 (no proof), Corollary 3.7.10 (Minkowski Inequality)(proof), Lemma 3.7.13 (proof), Theorem 3.7.15 (no proof), Remark 3.7.16 (see also the proof in the class notes).

CHAPTER 4: Definition 4.1.1, Theorem 4.1.5 (Fubini) (no proof), Remark 4.1.11, Applications in Section 4.2, Theorem 4.3.2 (no proof), Lemma 4.4.1 (proof), Lemma 4.4.2 (no proof), Definition 4.4.3 , Remark 4.4.4, Theorem 4.4.5 proof), Corollary 4.4.6 (proof only of the first part), Definition 4.4.7, Proposition 4.4.8 (proof, see also the alternative proof in the class notes).

For the preparation of the exam we advise to revise **all the bonus exercises** from exercises sheets of HS23 and the following additional exercises:

Serie 1: Ex 3,5,6.

Serie 2: Ex 1,2.

Serie 3: Ex 2.

Serie 4: Ex 3,4.

Serie 5: Ex 1,2.

Serie 6: Ex 2,3.

Serie 7: Ex 2,3,5 .

Serie 8: Ex 2,5.

Serie 9: Ex 3,5.

Serie 10: Ex 2,3,6.

Serie 11: Ex 2,3,4,5,6.

Serie 12: Ex 2,6.

Serie 13: Ex 2,4,7,8.