Lorenz Halbeisen

Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 11

---

51. Show that for any $n \geq 3$, A_n is generated by the set of all 3-cycles of the form $(1, 2, k)$, where $k \in \{3, \dots, n\}$.

52. (a) List all elements of A_4 and determine their orders.

    (b) Find the list of proper subgroups of A_4.

    (c) Decide which of these subgroups are normal subgroups.

53. (a) Show that if a group G has order pq^2, where p and q are two different prime numbers such that $p \nmid q^2 - 1$ or $q \nmid p - 1$, then G cannot be simple.

    (b) Show that a group of order 605 cannot be simple.

54. Show that a group of order 45 is always abelian.

    Hint: Use the fact that a group of order p^2, where p is prime, is always abelian.

55. (a) How many subgroups of order 2, 7 and 9 respectively a group of order 126 could have?

    (b) Can a group of order 126 be simple?