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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 2

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6. For $m \in \mathbb{Z}$, let $m\mathbb{Z} := \{mx : x \in \mathbb{Z}\}$.

   (a) What elements are in the sets $0\mathbb{Z}$, $17\mathbb{Z}$, $(-5)\mathbb{Z}$, $5\mathbb{Z}$.

   (b) Show that for any $m \in \mathbb{Z}$, the set $m\mathbb{Z}$ is the same as the set $(-m)\mathbb{Z}$.

   (c) Show that for any $m \in \mathbb{Z}$, addition is an associative binary operation on $m\mathbb{Z}$.

   (d) Show that for any $m \in \mathbb{Z}$, $(m\mathbb{Z}, +)$ is an abelian group.

7. On the set $(\mathbb{Q} \times \mathbb{Q})^* := (\mathbb{Q} \times \mathbb{Q}) \setminus \{(0,0)\}$ we define the binary operation "$\bullet$" as follows:

   $$(p_1, q_1)\bullet(p_2, q_2) := (p_1 p_2 - q_1 q_2,\ p_1 q_2 + q_1 p_2)$$

   Show that $\big((\mathbb{Q} \times \mathbb{Q})^*, \bullet\big)$ is an abelian group.

8. Let G be a group with neutral element e. Show that if for all $a \in G$ we have $aa = e$, then G is abelian.

9. Formulate a "right-version" of Proposition 1.2 and prove it.

10. Define an associative operation "$\diamond$" on set $\mathbb{Z}$ such that $(\mathbb{Z}, \diamond)$ has a right-neutral element and each element of $\mathbb{Z}$ has a left-inverse, but $(\mathbb{Z}, \diamond)$ is not a group.