Lorenz Halbeisen

GROUP THEORY (MODULE 210PMA208)
Department of Pure Mathematics

Week 7

---

31. Let G be a group.
    (a) Show that $Z(G) \leqslant G$.
    (b) Show that $Z(G) \trianglelefteq G$.
    (c) Show that $Z(G)$ is abelian.
    (d) Show that if $H \leqslant Z(G)$, then $H \trianglelefteq G$.

32. (a) What is the centre $Z(D_3)$ of D_3?
    (b) What is the centre $Z(D_4)$ of D_4?
    (c) For $n \geq 1$, what is the centre $Z(D_{2n+1})$ of D_{2n+1}?
    (d) For $n \geq 2$, what is the centre $Z(D_{2n})$ of D_{2n}?

33. What is the centre $Z\big(\mathrm{GL}(2)\big)$ of $\mathrm{GL}(2)$?

34. Let T be the tetrahedron-group, let ρ be an element of T of order 2 and let δ be an element of T of order 3. Further, let $H = \langle \rho \rangle$ and $K = \langle \delta \rangle$.
    (a) What is $H \cap K$?
    (b) Evaluate $|H|$, $|K|$, and $|HK|$.
    (c) Show that HK is not a subgroup of T.

35. Show that C_n is simple if and only if n is prime.