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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 9

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41. Let C be the cube-group and let N be the normal subgroup of C which is isomorphic to $C_2 \times C_2$.

    Show that C/N is isomorphic to S_3.

42. Let H be an abelian group and let $\varphi : G \to H$ be a homomorphism. Define a map $\psi : G \times G \to H$ by
$$\psi(g_1, g_2) = \varphi(g_1)\varphi(g_2)^{-1}\,.$$
    Prove that ψ is a homomorphism.

43. Let $D_3 = \{\iota, \rho, \rho^2, \sigma_1, \sigma_2, \sigma_3\}$, where ρ is a rotation through $2\pi/3$ and $\sigma_1, \sigma_2, \sigma_3$ are the three reflections. Define the homomorphism $\varphi : D_3 \to \big(\{1, -1\}, \cdot\big)$ by $\varphi(g) = 1$ if g is a rotation, and $\varphi(g) = -1$ if g is a reflection. Further, let $\psi : D_3 \times D_3 \to \big(\{1, -1\}, \cdot\big)$ be defined as in Question 42.

    List the elements in $\ker(\psi)$.

44. Let $\varphi : G \to H$ be a homomorphism.

    (a) Prove by induction that, for all positive integers k and for all $g \in G$, we have $\varphi(g^k) = \varphi(g)^k$.

    (b) Deduce that if g has finite order k, then the order of $\varphi(g)$ divides k.

    (c) Show that if φ is injective, then the order of $\varphi(g)$ is equal to k.

45. Determine the elements of $\mathrm{Aut}(C_4)$ and write down its Cayley table.