Dr Halbeisen*

Modules 110PMA003 & 110PMA107

Department of Pure Mathematics

Week 11, 2001

The pdf-file you may download from
`http://www.math.berkeley.edu/~halbeis/4students/zero.html`

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46. (a) Evaluate $\lim\limits_{x\to+\infty} \frac{\ln(x)}{x}$ by using the rule of Bernoulli-de l'Hôpital.

    (b) Evaluate $\lim\limits_{x\to+\infty} \sqrt[x]{x}$.

    Hint: $\sqrt[x]{x} = x^{\frac{1}{x}} = \left(e^{\ln(x)}\right)^{\frac{1}{x}} = e^{\frac{\ln(x)}{x}}$.

47. (a) Evaluate $\lim\limits_{x\to0} \frac{\ln(1+x)}{x}$ by using the rule of Bernoulli-de l'Hôpital.

    (b) Using part (a), show that for small values of x we have $\ln(1+x) \approx x$.

    (c) For natural numbers n, evaluate $\lim\limits_{n\to+\infty} \left(1+\frac{x}{n}\right)^n$ and give the exact value of $\lim\limits_{n\to+\infty} \left(1+\frac{1}{n}\right)^n$.

    Hint: Notice that $\ln\left(\lim\limits_{n\to+\infty}(1+\frac{x}{n})^n\right) = n \lim\limits_{n\to+\infty} \ln\left(1+\frac{x}{n}\right)$.

48. (a) Use integration by parts to determine

$$\int x\,\cos(3x)\,dx\,.$$

    (b) Calculate the area under the curve $y = x\,\cos(3x)$ between $x = 0$ and $x = \pi$.

49. Use integration by parts to calculate

$$\int_{-\infty}^{0} e^x\,\sin(x)\,dx\,.$$

50. Determine $\int_0^{\frac{\pi}{4}} \tan(x)\,dx\,.$

    Hint: Remember that $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

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Office hours (Room 1007): Monday 1 pm–2 pm, Wednesday 2 pm–3 pm