Dr Halbeisen*

Modules 110PMA003 & 110PMA107

Department of Pure Mathematics

Week 2, 2001

The pdf-file you may download from `http://www.math.berkeley.edu/~halbeis/4students/zero.html`

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Thursday, 11th of October 2001.

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5. Use the fact that $\sum_{n=0}^{\infty} \frac{1}{2^n} = 2$ to evaluate the following. (Give the *exact* values, or in other words, *don't* use a calculator.)
   (a) $\sum_{n=0}^{\infty} \frac{7}{9 \cdot 2^n}$
   (b) $\sum_{n=1111}^{\infty} \frac{1}{2^n}$
   (c) $\sum_{n=0}^{1110} \frac{1}{2^n}$
   (d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{2}^n}$
6. (a) Write $z = \frac{(7-i)}{(4+i3)}$ in the form $(a+ib)$, where $a, b \in \mathbb{R}$.
   (b) For $z = (-12 + i5)$ determine $|z|$, $\overline{z}$, $-z$, $-\overline{z}$ and $\overline{-z}$, and show these numbers in an Argand diagram.
7. On an Argand diagram show the set of complex numbers for which
$$|z| \leq 1 \text{ and } \operatorname{Im}(z) \geq 0\,.$$
8. Let $E(z) := \sum_{n=0}^{6} \frac{z^n}{n!}$, thus, the function $E(z)$ is an approximation for e^z. Now, compute (using a calculator) the following:
   (a) $E\big(i\frac{\pi}{2}\big)$, $\big|E\big(i\frac{\pi}{2}\big)\big|$
   (b) $\sqrt{2} \cdot E\big(i\frac{\pi}{4}\big)$, $\big|\sqrt{2} \cdot E\big(i\frac{\pi}{4}\big)\big|$
   (c) $E\big(-i\frac{\pi}{4}\big)$, $\big|E\big(-i\frac{\pi}{4}\big)\big|$
9. Show the following:
   (a) $(-\frac{1}{2} + i\frac{\sqrt{3}}{2})^3 = (-\frac{1}{2} - i\frac{\sqrt{3}}{2})^3 = 1^3 = 1$.
   (b) $(\sqrt{3} + i\sqrt{3})^4 = (\sqrt{3} - i\sqrt{3})^4 = (-\sqrt{3} + i\sqrt{3})^4 = (-\sqrt{3} - i\sqrt{3})^4 = -36$.

   Thus, $z^3 = 1$ has at least 3 solutions and $z^4 = -36$ has at least 4 solutions.

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Office hours: Monday 1.00 p.m.–2.00 p.m., Wednesday 2.00 p.m.–3.00 p.m.