12208. *Proposed by Gregory Galperin, Eastern Illinois University, Charleston, IL, and Yury J. Ionin, Central Michigan University, Mount Pleasant, MI.* (In memory of John Horton Conway, 1937–2020.) Three wise women, Alice, Beth, and Cecily, sit around a table. A card with a positive integer on it is attached to each woman's forehead, so she can see the other two numbers but not her own. The women know that one of the three integers is equal to the sum of the other two. The same question, "Can you determine the number on your forehead?," is addressed to the wise women in the following order: Alice, Beth, Cecily, Alice, Beth, Cecily, ... The answer is either "No" or "Yes, the number is ___," and the other wise women hear the answer. The questioning ends as soon as the positive answer is obtained. (Assume that the women are logical and honest, they all know this, they all know that they all know this, and so on.)

(a) Prove that whichever numbers are assigned to the wise women, an affirmative answer is obtained eventually.

(b) Suppose that Alice's second answer is "Yes, the number is 50." Determine the numbers assigned to Beth and Cecily.

(c) Suppose the numbers assigned to Alice, Beth, and Cecily are 1492, 1776, and 284, respectively. Determine who will give the affirmative answer and how many negative answers she will give before that.