Solution of Riddle 12224

The Logic Coffee Circle*

Abstract

We solve the following riddle:

12224. *Proposed by Cherng-tiao Perng, Norfolk State University, Norfolk, VA.* Let ABC be a triangle, with D and E on AB and AC, respectively. For a point F in the plane, let DF intersect BC at G and let EF intersect BC at H. Furthermore, let AF intersect BC at I, let DH intersect EG at J, and let BE intersect CD at K. Prove that I, J, and K are collinear.

Solution

We consider the problem in the real projective plane. By a projective transformation we may assume that, in projective coordinates, $A = (0, 1, 0)$, $B = (0, 0, 1)$, and the intersection of BC with DE is the point $(1, 0, 0)$. This leads to the situation in Figure 1 (or to a similar situation).

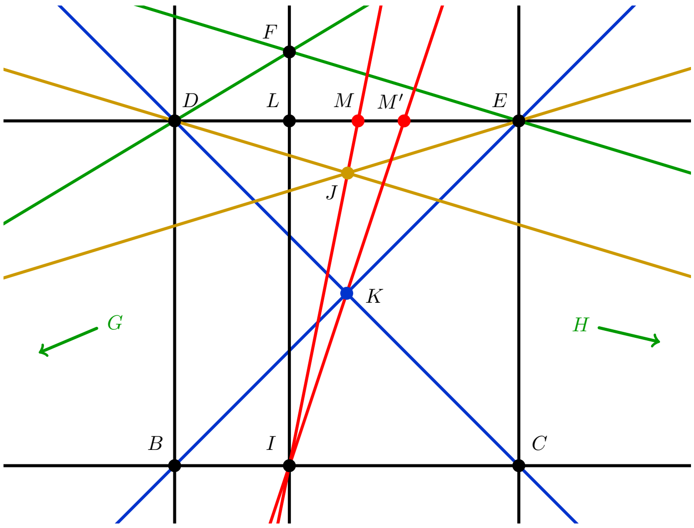

Figure 1

Let FI intersect DE at L, let JI intersect DE at M, and let KI intersect DE at M'. By the intercept theorem, we have that

$$\frac{\overline{IB}}{\overline{IC}} = \frac{\overline{LD}}{\overline{LE}} = \frac{\overline{IG}}{\overline{IH}} = \frac{\overline{ME}}{\overline{MD}} \quad \text{and} \quad \frac{\overline{IB}}{\overline{IC}} = \frac{\overline{M'E}}{\overline{M'D}}.$$

Hence, $\frac{\overline{ME}}{\overline{MD}} = \frac{\overline{M'E}}{\overline{M'D}}$ which implies that $M = M'$.

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