

# CORRECTIONS AND IMPROVEMENTS

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## Chapter 1

- page 12, line -18 f.     The elements of a theory are the axioms of the theory, **which are called non-logical axioms**. **In general, a non-logical axiom is just a formula which is not a logical axiom**. Notice that ~~non-logical axioms are sentences (i.e., formulae without free variables)~~.
- page 13, line 3     ...with  $\text{free}(\varphi) = \{x\}$
- page 13, line -12     and for variables  $\nu$  ~~which do not occur free in any non-logical axiom~~:
- page 13, line -7 f.     ~~It is worth mentioning that the restriction on  $(\forall)$  is not essential, but will simplify certain proofs (e.g., the proof of the DEDUCTION THEOREM 2.1).~~
- page 16, line 15     ~~In Appendix 17~~ **At the end of the book ...**

## Chapter 2

- page 19, line -7 f.     ...and  $\Phi + \psi \vdash \varphi$ , **where in the formal proof of  $\varphi$  from  $\Phi + \psi$ , Generalisation was not applied to variables which occur free in  $\psi$** , then...
- page 20, line 17     ...does not occur free in  $\psi$ .
- page 22, line 1      $\Phi + \{\varphi\}$
- page 25, line -7     (PROOF BY CONTRA**POSITION**)
- page 28, line 5     instance of **L12**
- page 28, line -2      $\forall \nu \varphi \circ \psi$
- page 30, line 1     ... for some **sentence**  $\varphi$  ...
- page 33, line 1     *We first show  $\psi$  ... This proves  $\varphi$ .*

## Chapter 3

- page 39, line 7      $M_2 \models \neg\varphi_1 \wedge \varphi_2$

## Chapter 4

- page 51, line 1 f.     ... such that  $\sigma_m \in \bar{T}$ . ~~if no such  $m$  exists, we set  $m = 0$~~
- page 51, line 3     ... contradicting  $\sigma_m \in \bar{T}$ ; **notice that  $\text{Con}(T + T_0)$** . ~~(respectively  $T \not\vdash \sigma_0$  in the case of  $m = 0$ )~~
- page 51, line 9     ...  $T_0 = [\neg\sigma_0]$  is ...

## Chapter 5

page 54, line 1

 $\# \tau_0, \dots$ 

page 60, line 8

 $\sigma \in \tilde{T} \iff \mathbf{M} \models \sigma$ **Chapter 8**

page 84, line 16

... no common divisor **greater than 1**.**Chapter 9**

page 96, line 1

... strictly increasing function with  $G(0) > 1$  which is ...

page 101, line -15

...  $2\# \wedge . 3\# \psi_0 . 5\# \psi_1$ 

page 104, line -16

 $(\text{var}(c_k) \wedge c_k \neq v \rightarrow c'_{k+\text{lh}(c')} = c_k)$ **Chapter 10**

page 118, line -10

 $\text{prv}_T^R(x)$ 

page 118, line -9

 $\text{prv}_T^R(\ulcorner \sigma \urcorner)$ **Chapter 13**

page 155, line 9

...  $\forall z(z \in x \rightarrow z \in y)$ 

page 155, line -15

...  $\{x\}$ , **where  $\{x\}$  denotes the set which contains the single element  $x$** .

page 159, line -2

... with domain  $\alpha$ , **for some ordinal number  $\alpha$** , then...

page 162, line 1

 $\forall x(x \neq \emptyset \rightarrow \exists y(y \in x \wedge (y \cap x = \emptyset)))$ **Chapter 14**

page 179, line -10

 $\ulcorner \forall v_j \varphi \urcorner := \langle 7, j, \ulcorner \varphi \urcorner \rangle$ 

page 186, line 6

... from every **non-empty** set.**Chapter 17**

page 203, line -7

 $\forall x \forall y(x + y = y + x)$ 

page 204, line 6

 $\forall x \forall y \forall z(x < y \wedge 0 < z \rightarrow \dots)$ 

page 204, line -3

... of the form  $\langle 0, y \rangle$ .

page 205, line 13

 $\langle x_0, y_0 \rangle < \langle x_1, y_1 \rangle \iff y_0 + x_1 < y_1 + x_0$ 

page 205, line 14

 $z = \langle x, y \rangle$  if  $\langle x, y \rangle > \langle 0, 0 \rangle$ 

page 208, line -7

...  $+ |a_k^m - a_k^n| + \dots$ 

page 209, line 3

 $N := \max\{N_0, N_1, \lceil \frac{3}{\delta} \rceil\}$ , **where  $\lceil \frac{3}{\delta} \rceil$  is the least integer bigger than or equal to  $\frac{3}{\delta}$** 

page 209, line 10

Since  $b_k > \delta$ , ...

page 214, line -4

 $-[-n \cdot a_{k-n}]$  otherwise.

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## MINOR CORRECTIONS AND IMPROVEMENTS

- page 9, line -12      ... most basic **formulae** we have, ...
- page 11, line -22      ... arbitrary ~~first-order~~ formulae
- page 14, line -12      ~~instance of~~ PA<sub>3</sub>
- page 14, line -7      ~~instance of~~ PA<sub>2</sub>
- page 14, line -1       $\varphi_9 \rightarrow (\varphi_{10} \rightarrow \dots$
- page 15, line 1       $\varphi_{10} \rightarrow (\varphi_{10} \wedge \varphi_9)$       from  $\varphi_{11}$  and  $\varphi_9$  by...
- page 15, line 2      Commutativity and associativity of  $\wedge$  and  $\vee$  (**up to logical equivalence**)...
- page 16, line 9      from  $\varphi_{12}$  and  $\varphi_{10}$  by...
- page 17, line -11      Prove (K), (L.0), and (R) **from the tautologies list at book's end** first.
- page 22, line 9      from  $\varphi_{12}$  and  $\varphi_{10}$  by...
- page 36, line 6      In other words, ...
- page 40, line 5 ff.      replace  $\varphi_0$  by  $\varphi$ .
- page 43, line -13      ...whether ~~the~~...
- page 50, line -22      ... as an initial **segment**.
- page 96, line 1      ... be ~~an~~ unary ...
- page 137, line -8       $\mathcal{L}_{PrA} = \{0, s, +, \}$