ETH Zürich, Basisprüfung
Analysis I/II D-BAUG Winter 2013
Dr. Meike Akveld

Important information

- Duration of the exam
  - Basisprüfung Analysis I/II: Exercises 1–10, 240 minutes
  - Semesterkurs Analysis I: Exercises 1–5, 120 minutes
  - Semesterkurs Analysis II: Exercises 6–10, 120 minutes
- Permitted aids; 15 sheets DIN A4 (= 30 pages) self-authored summary (for the Analyis II exam only 10 sheets DIN A4 (= 20 pages)); no calculator!
- All answers must be justified and the appraoch of the solution must be clearly illustrated. Correct, but unjustified solutions will not give any points. If you make use of a theorem from the lecture, you have to specify precisely which theorem is used.
- All exercises carry the same weight.

$* * *$ Lots of success! $* * *$

Bitte wenden!

1. **a)** [**3 P**] Determine the absolute value, the argument and the real and imaginary part of the following complex number

$$w = \frac{(1-\sqrt{3}i)^4}{1-i} \cdot \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) .$$

b) [**3 P**] Make a sketch in the complex plane of the domain determined by all complex numbers that satisfy the following two conditions

$$\left|\frac{z}{1-i}\right| = \sqrt{8} \text{ and } 0 \le \arg\frac{z}{i} \le \frac{3}{4}\pi .$$

2. **a)** [**3 P**] Determine the following limit

$$\lim_{x\to 0}\left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x}\right) .$$

b) [**3 P**] Determine a fourth-order Taylor series expansion of the function $f(x) = \cos(x)^2$ around $x_0 = 0$.

3. [**6 P**] Consider the two circles C_1 and C_2 with radii r_1 and r_2 respectively, which both pass through the origin and whose centers both lie on the positive y-axis. The line g passes through the origin, has a positive slope and intersects the two circles in the the points I_1 and I_2. View these two points as two diagonal vertices of a rectangle, whose sides are parallel to the coordinate axes. Determine the slope of the line g so that this rectangle has maximal area and compute this area.

4. Compute the following integrals:

a) [**1 P**] $\int_0^{\frac{\pi}{6}} x^2 \sin 3x \, dx$.

b) [**2 P**] $\displaystyle\int \frac{4\ln(\frac{1}{\tan x})}{\sin x \cos x} \, dx$.

c) [**3 P**] $\displaystyle\int \frac{4x^3 + 4x^2 + 6x - 1}{2x^2 - 2x + 1} \, dx$.

5. [**6 P**] Determine the general solution of the following ordinary differential equation

$$y'''(x) + 2y'(x) = x + e^x .$$

Siehe nächstes Blatt!

6. Consider the surface S :

$$S : (y+z)^2 + (z-x)^2 = 16\ ,$$

a) [**3 P**] Determine the set of all points on S for which the normal to the surface is parallel to the yz-plane and describe this set geometrically.

b) [**3 P**] Compute, for the points you found in a), the equation of the tangent plane to S.

7. [**6 P**] Consider the function $f(x, y) = x^3 - x^2 - y^3 - y^2 + 1$. Determine the global extrema of f over the region B (see Figure 1). Note that the boundary of B is included in the region B.

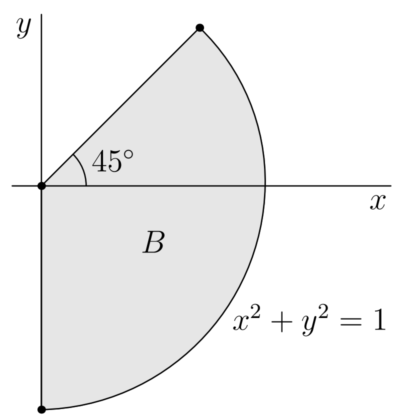

Figure 1: Exercise 7.

8. Consider the finite solid K in the first octant, which is bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = y$.

a) [**2 P**] Make a sketch of K.

b) [**4 P**] Determine the volume of K.

Bitte wenden!

9. [**6 P**] The surface S consists of two parts i.e. $S = S_1 \cup S_2$, where

$$S_1 = \left\{(x,y,z) | y \geq 0, z \leq 1, z = x^2 + y^2\right\}$$

and

$$S_2 = \left\{(x,y,z) | y = 0, 1 \geq z \geq x^2\right\}$$

(see Figure 2).
Determine the integral $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x,y,z) = (xz - xy, xy - yz, yz - xz)$ and the normal of S is pointing outwards.

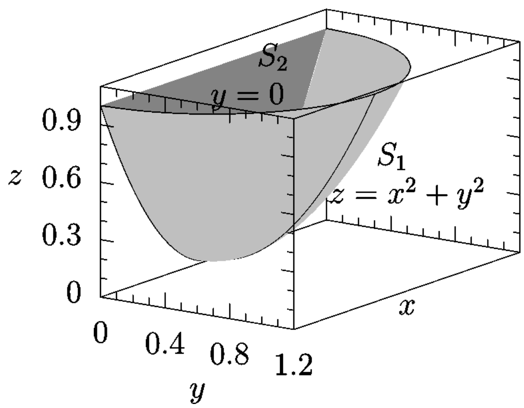

Figure 2: Exercise 9

10. [**6 P**] Determine – using the method of separation of variables – a solution $u(x,t)$ of the following boundary value problem:

$$\begin{cases} u_{xx} &= u_t - u \quad \text{for } 0 < x < \pi \text{ and } 0 < t \\ u(0,t) &= 0 \\ u(\pi,t) &= 0 \\ u(x,0) &= \cos(2x)\sin(x). \end{cases}$$

Hint: You may use the following trigonometric identities without prof:

$$\begin{aligned} \cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right) \\ \cos\alpha\sin\beta &= \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right) \\ \sin\alpha\sin\beta &= \frac{1}{2}\left(-\cos(\alpha+\beta) + \cos(\alpha-\beta)\right). \end{aligned}$$