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A Projection Result for Semimartingales

by

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A Projection Result for Semimartingales

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Abstract: Let X be a semimartingale and Θ the space of all predictable X-integrable processes ϑ such that $G(\vartheta) := \int \vartheta \, dX$ is in the space $\mathcal{S}^2$ of semimartingales. Assume that X is special and has the form $X = X_0 + M + \int \alpha \, d\langle M \rangle$. We show that for every fixed $T > 0$, the space $G_T(\Theta)$ of stochastic integrals is closed in $\mathcal{L}^2$ if the process $\int \alpha^2 \, d\langle M \rangle$ is bounded on $[0, T]$ and has jumps strictly bounded above by 1. This allows us to solve a quadratic optimization problem arising in financial mathematics.

Key words: semimartingales, stochastic integrals, projection theorem, mean-variance tradeoff, financial mathematics

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0. Introduction

If M is a square-integrable martingale, then by its very construction, the stochastic integral with respect to M is an isometry. For every fixed $T > 0$, the space of stochastic integrals

$$\left\{ \int_0^T \vartheta_s \, dM_s \,\middle|\, \int \vartheta \, dM \text{ is a square-integrable martingale} \right\}$$

is therefore a closed subspace of $\mathcal{L}^2$. In this paper, we extend this result to a certain class of $\mathbb{R}^d$-valued *semimartingales.* For ease of exposition, we formulate the results in the introduction only for $d = 1$. We assume that X is in $\mathcal{S}^2_{\mathrm{loc}}$ and has a canonical decomposition of the form

$$X = X_0 + M + \int \alpha \, d\langle M \rangle.$$

The process

$$\widehat{K}_t := \int_0^t \alpha_s^2 \, d\langle M \rangle_s \qquad , \qquad 0 \le t \le T$$

is called the *mean-variance tradeoff process* for X. Our main result then states that if $\widehat{K}_T$ is P-a.s. bounded and if

$$\text{(0.1)} \qquad \sup \left\{ \widehat{K}_\tau - \widehat{K}_{\tau-} \,\middle|\, \tau \text{ is stopping time} \le T \ P\text{-a.s.} \right\} \le b < 1 \qquad P\text{-a.s.}$$

for some constant b, then the space

$$\left\{ \int_0^T \vartheta_s \, dX_s \,\middle|\, \int \vartheta \, dX \text{ is a semimartingale in } \mathcal{S}^2 \right\}$$

is also *closed* in $\mathcal{L}^2$. This is rather remarkable since in contrast to the martingale case, stochastic integration with respect to a semimartingale is in general not an isometry. We point out that recent independent work by P. Monat and C. Stricker has shown that condition (0.1) is actually unnecessary; see Monat/Stricker [8,9] for details. On the other hand, a counterexample in section 3 illustrates that boundedness of $\widehat{K}$ is in general indispensable. As an immediate application, we obtain an existence result for a quadratic optimization problem arising in financial mathematics.

1. Preliminaries

Let $(\Omega, \mathcal{F}, P)$ be a probability space with a filtration $\mathbb{F} = (\mathcal{F}_t)_{0 \le t \le T}$ satisfying the usual conditions of right-continuity and completeness, where $T > 0$ is a fixed and finite time horizon. For unexplained notation, terminology and results from martingale theory, we refer to Dellacherie/Meyer [3] and Jacod [6]. Without special mention, all processes will be defined for $t \in [0, T]$. Let X be an $\mathbb{R}^d$-valued semimartingale in $\mathcal{S}^2_{\mathrm{loc}}$; for the canonical decomposition

$$X = X_0 + M + A,$$

this means that $M \in \mathcal{M}^2_{0,\mathrm{loc}}$ and that the variation of the predictable finite variation part A^i of X^i is locally square-integrable for each i. We can and shall choose versions of M and A such that M^i and A^i are RCLL for each i. We shall assume that for each i,

$$A^i \ll \langle M^i \rangle \qquad \text{with predictable density } \alpha^i. \tag{1.1}$$

Throughout the sequel, we fix a predictable locally integrable increasing RCLL process B null at 0 such that $\langle M^i \rangle \ll B$ for each i. Since this implies $\langle M^i, M^j \rangle \ll B$ for all i, j, we can define the predictable matrix-valued process σ by

$$\sigma^{ij}_t := \frac{d\langle M^i, M^j \rangle_t}{dB_t} \qquad \text{for } i, j = 1, \ldots, d.$$

We also define the predictable $I\!\!R^d$-valued process γ by

$$\gamma^i_t := \alpha^i_t \sigma^{ii}_t \qquad \text{for } i = 1, \ldots, d,$$

so that for each i,

$$A^i_t = \int_0^t \gamma^i_s \, dB_s. \tag{1.2}$$

Definition. The space $L^2(M)$ consists of all predictable $I\!\!R^d$-valued processes ϑ such that

$$E\left[\int_0^T \vartheta^*_s \sigma_s \vartheta_s \, dB_s\right] < \infty, \tag{1.3}$$

where * denotes transposition. The space $L^2(A)$ consists of all predictable $I\!\!R^d$-valued processes ϑ such that

$$E\left[\left(\int_0^T \left|\vartheta^*_s \gamma_s\right| dB_s\right)^2\right] < \infty.$$

Finally, we set $\Theta := L^2(M) \cap L^2(A)$.

Definition. We say that X *satisfies the structure condition (SC)* if there exists a predictable $I\!\!R^d$-valued process $\widehat{\lambda}$ such that

$$\sigma_t \widehat{\lambda}_t = \gamma_t \qquad P\text{-a.s. for all } t \in [0, T] \tag{1.4}$$

and

$$\widehat{K}_t := \int_0^t \widehat{\lambda}^*_s \gamma_s \, dB_s < \infty \qquad P\text{-a.s. for all } t \in [0, T].$$

We then choose an RCLL version of $\widehat{K}$ and call it the *mean-variance tradeoff process* of X. Note that these definitions imply that $\widehat{\lambda} \in L^2_{\mathrm{loc}}(M)$ and

$$\widehat{K} = \left\langle \int \widehat{\lambda} \, dM \right\rangle.$$

Condition (SC) is naturally satisfied in most situations arising in financial mathematics; see Schweizer [14]. For $d=1$, we can choose $B := \langle M \rangle$ and $\widehat{\lambda} := \alpha = \gamma$; condition (SC) then follows from (1.1) and the assumption that $\alpha \in L^2_{\mathrm{loc}}(M)$, and $\widehat{K} = \int \alpha^2 \, d\langle M \rangle$.

For any $\vartheta \in \Theta$, the stochastic integral process $G(\vartheta) := \int \vartheta \, dX$ is well-defined and a semimartingale in $\mathcal{S}^2$ with canonical decomposition

$$G(\vartheta) = \int \vartheta \, dM + \int \vartheta^* dA.$$

For our purposes, it is more convenient to use an alternative description of the space Θ. If we denote by $L(X)$ the set of all $I\!R^d$-valued X-integrable predictable processes, then we have (as in Schweizer [13])

Lemma 1. *If X satisfies (1.1), then*

$$\Theta = \left\{ \vartheta \in L(X) \middle| \int \vartheta \, dX \in \mathcal{S}^2 \right\} =: \Theta'.$$

If in addition X satisfies (SC) and $\widehat{K}_T$ is bounded, then $\Theta = L^2(M)$.

Proof. Since the variation of $\int \vartheta^* dA$ is given by $\int |\vartheta^* \gamma| \, dB$, it is clear that Θ' contains $L^2(M) \cap L^2(A)$. Conversely, X is special and $\int \vartheta \, dX$ is special for any $\vartheta \in \Theta'$; hence $\int \vartheta \, dM$ and $\int \vartheta^* dA$ both exist in the usual sense by Théorème 2 of Chou/Meyer/Stricker [2], and $\int \vartheta \, dX \in \mathcal{S}^2$ thus implies that $\vartheta \in L^2(M) \cap L^2(A)$. Finally,

$$\int_0^T |\vartheta_s^* \gamma_s| \, dB_s \le \int_0^T (\vartheta_s^* \sigma_s \vartheta_s)^{\frac{1}{2}} \left(\widehat{\lambda}_s^* \sigma_s \widehat{\lambda}_s \right)^{\frac{1}{2}} dB_s \le \left(\widehat{K}_T \right)^{\frac{1}{2}} \left(\int_0^T \vartheta_s^* \sigma_s \vartheta_s \, dB_s \right)^{\frac{1}{2}}$$

shows that $L^2(M) \subseteq L^2(A)$ if $\widehat{K}_T$ is bounded.

q.e.d.

2. The main result

Let us now study in more detail the space $G_T(\Theta)$ of stochastic integrals.

Lemma 2. *Suppose that τ, τ' are stopping times with $\tau \le \tau' \le T$ P-a.s. and*

(2.1) $$\widehat{K}_{\tau'} - \widehat{K}_{\tau} \le c < 1 \qquad P\text{-a.s. for some constant } c.$$

Then there exists a constant $C \in (0, \infty)$, depending only on c, such that for every $\vartheta \in \Theta$,

(2.2) $$E\left[\left(\int_\tau^{\tau'} \vartheta_s \, dM_s \right)^2 \right] \le C E\left[\left(\int_0^{\tau'} \vartheta_s \, dX_s \right)^2 \right].$$

Proof. Choose $\varepsilon > 0$ such that $(1+\varepsilon)c < 1$. Write

$$\int_\tau^{\tau'} \vartheta_s\, dM_s = \int_\tau^{\tau'} \vartheta_s\, dX_s - \int_\tau^{\tau'} \vartheta_s^*\, dA_s$$
$$= \int_0^{\tau'} \vartheta_s\, dX_s - E\left[\int_0^{\tau'} \vartheta_s\, dX_s \middle| \mathcal{F}_\tau\right] - \left(\int_\tau^{\tau'} \vartheta_s^*\, dA_s - E\left[\int_\tau^{\tau'} \vartheta_s^*\, dA_s \middle| \mathcal{F}_\tau\right]\right)$$

and use the inequality

$$(u-v)^2 \le \left(1+\frac{1}{\varepsilon}\right) u^2 + (1+\varepsilon)v^2$$

to obtain

$$\text{(2.3)}\; E\left[\left(\int_\tau^{\tau'} \vartheta_s\, dM_s\right)^2\right] \le \left(1+\frac{1}{\varepsilon}\right) E\left[\operatorname{Var}\left[\int_0^{\tau'} \vartheta_s\, dX_s \middle| \mathcal{F}_\tau\right]\right]$$
$$+ (1+\varepsilon) E\left[\operatorname{Var}\left[\int_\tau^{\tau'} \vartheta_s^*\, dA_s \middle| \mathcal{F}_\tau\right]\right]$$
$$\le \left(1+\frac{1}{\varepsilon}\right) E\left[\left(\int_0^{\tau'} \vartheta_s\, dX_s\right)^2\right] + (1+\varepsilon) E\left[\left(\int_\tau^{\tau'} \vartheta_s^*\, dA_s\right)^2\right].$$

By (1.2), (1.4) and the Cauchy-Schwarz inequality,

$$\text{(2.4)} \qquad \left(\int_\tau^{\tau'} \vartheta_s^*\, dA_s\right)^2 \le \int_\tau^{\tau'} \vartheta_s^* \sigma_s \vartheta_s\, dB_s \int_\tau^{\tau'} \widehat{\lambda}_s^* \sigma_s \widehat{\lambda}_s\, dB_s = \left(\widehat{K}_{\tau'} - \widehat{K}_\tau\right) \int_\tau^{\tau'} \vartheta_s^* \sigma_s \vartheta_s\, dB_s,$$

and so we conclude from (2.1) that

$$E\left[\left(\int_\tau^{\tau'} \vartheta_s^*\, dA_s\right)^2\right] \le cE\left[\int_\tau^{\tau'} \vartheta_s^* \sigma_s \vartheta_s\, dB_s\right] = cE\left[\left(\int_\tau^{\tau'} \vartheta_s\, dM_s\right)^2\right].$$

Inserting this in (2.3) and rearranging yields (2.2), with

$$C = \frac{1+\frac{1}{\varepsilon}}{1-(1+\varepsilon)c}.$$

q.e.d.

Theorem 3. *Assume that $\widehat{K}_T$ is P-a.s. bounded by a constant and that*

$$\text{(2.5)} \qquad \sup\left\{\widehat{K}_\tau - \widehat{K}_{\tau-} \,\middle|\, \tau \text{ is stopping time} \le T \;P\text{-a.s.}\right\} \le b < 1 \qquad P\text{-a.s.}$$

for some constant b. Then $G_T(\Theta)$ is closed in $\mathcal{L}^2$.

Proof. Thanks to the boundedness of $\widehat{K}_T$ and (2.5), we can find $N \in I\!N$ and stopping times $0 = \tau_0 \leq \tau_1 \leq \ldots \leq \tau_N = T$ P-a.s. such that

$$\widehat{K}_{\tau_j} - \widehat{K}_{\tau_{j-1}} \leq c < 1 \qquad P\text{-a.s. for } j = 1, \ldots, N \text{ and some constant } c.$$

Now suppose that $\big(G_T(\vartheta^m)\big)_{m \in I\!N}$ converges in $\mathcal{L}^2$ to some limit Y. Applying Lemma 2 with $\tau' := T$ and $\tau := \tau_{N-1}$ shows that

$$\left(\int_{\tau_{N-1}}^{T} \vartheta^m_s \, dM_s \right)_{m \in I\!N} \qquad \text{is a Cauchy sequence in } \mathcal{L}^2.$$

Hence $\big(\vartheta^m I_{]\!]\tau_{N-1},T]\!]}\big)_{m \in I\!N}$ is a Cauchy sequence in $L^2(M)$ and thus converges to $\psi^N I_{]\!]\tau_{N-1},T]\!]}$ for some $\psi^N \in L^2(M)$. Since $\widehat{K}_T$ is bounded, (2.4) yields

$$E\left[\left(\int_{\tau}^{\tau'} \vartheta_s \, dX_s \right)^2 \right] \leq 2 \left(1 + \|\widehat{K}_T\|_\infty \right) E\left[\left(\int_{\tau}^{\tau'} \vartheta_s \, dM_s \right)^2 \right]$$

for every $\vartheta \in \Theta$ and all stopping times $\tau \leq \tau' \leq T$ P-a.s. This implies that

$$\int_{\tau_{N-1}}^{T} \vartheta^m_s \, dX_s \ \text{ converges to } \int_{\tau_{N-1}}^{T} \psi^N_s \, dX_s \ \text{ in } \mathcal{L}^2,$$

and therefore $\big(G_{\tau_{N-1}}(\vartheta^m)\big)_{m \in I\!N}$ converges in $\mathcal{L}^2$ to $Y - \int_{\tau_{N-1}}^{T} \psi^N_s \, dX_s$. Iterating this argument shows that

$$Y = \int_0^T \vartheta^\infty_s \, dX_s$$

with

$$\vartheta^\infty := \sum_{j=1}^{N} \psi^j I_{]\!]\tau_{j-1},\tau_j]\!]},$$

and since ϑ^∞ is clearly in $L^2(M) = \Theta$, the assertion follows.

q.e.d.

Remarks. 1) A simple modification of the proof of Lemma 2 yields the inequalities

$$E\left[\left(\int_{\tau}^{\tau'} \vartheta_s \, dM_s \right)^2 \right] \leq C E\left[\left(\int_{\tau}^{\tau'} \vartheta_s \, dX_s \right)^2 \right] \leq 2C(1+c) E\left[\left(\int_{\tau}^{\tau'} \vartheta_s \, dM_s \right)^2 \right].$$

This provides the intuition behind the closedness result in Theorem 3: under the assumptions made there,

$$\left\| \int_0^T \vartheta_s \, dX_s \right\|_{\mathcal{L}^2} \qquad \text{and} \qquad \left\| \int_0^T \vartheta_s \, dM_s \right\|_{\mathcal{L}^2} = \|\vartheta\|_{L^2(M)}$$

are essentially equivalent norms on Θ, and so the semimartingale case considered here is not too far away from the martingale case. For a proof that the above two norms are actually equivalent, see Monat/Stricker [9].

2) It is interesting to note that the conditions of Theorem 3 are exactly the same as those guaranteeing the existence of a strong F-S decomposition for $\mathcal{F}_T$-measurable random variables $H \in \mathcal{L}^2$; see Schweizer [13]. We also point out that the above proof is analogous to the argument for the discrete-time case treated in Schweizer [12].

3) After this paper was submitted, we learnt from C. Stricker that he and P. Monat had independently also proved the closedness of $G_T(\Theta)$, even without assuming condition (2.5). Their argument rests on showing that the strong F-S decomposition of a square-integrable random variable exists and is unique and continuous; see Monat/Stricker [8]. In a subsequent paper, Monat/Stricker [9] then showed how to modify the direct argument of the present paper in order to eliminate assumption (2.5). In both cases, the essential step is to use the predictability of ϑ, A and $\widehat{K}$ in a suitable way.

3. Applications and examples

Apart from condition (2.5), Theorem 3 is the best possible result. The following example due to W. Schachermayer shows that $G_T(\Theta)$ need not be closed in $\mathcal{L}^2$ if $\widehat{K}_T$ is unbounded. For simplicity, we formulate the example in discrete time; choosing piecewise constant RCLL processes and a piecewise constant right-continuous filtration immediately yields a continuous-time version. For a similar example with a continuous process X, see Monat/Stricker [8].

Example. Let S, U be independent with U uniform on $[0,1]$ and the distribution of S nondegenerate with finite second moment. Given U, the random variable V takes the values ± 1 with respective probabilities $U^2, 1-U^2$. Take $T = 2$ and define the discrete-time process $(X_k)_{k=0,1,2}$ by setting $X_0 = 0$, $X_1 = S$ and $X_2 = (S+U)V^+$. The filtration $(\mathcal{F}_k)_{k=0,1,2}$ is given by $\mathcal{F}_0 = \sigma(U)$, $\mathcal{F}_1 = \sigma(U,S)$ and $\mathcal{F}_2 = \sigma(U,S,V)$. Then we get

$$\widehat{K}_1 = \frac{(E[S])^2}{\mathrm{Var}[S]},$$
$$\widehat{K}_2 = \frac{(U^3 + SU^2 - S)^2}{U^2(U+S)^2 - U^5(U+2S)},$$

and as U approaches 0, the last ratio tends to infinity so that $\widehat{K}_2$ is unbounded in ω.

Now consider the sequence of predictable processes

$$\vartheta^n = \frac{1}{U} I_{\{U \geq \frac{1}{n}\}}.$$

Then

$$G_1(\vartheta^n) = \frac{S}{U} I_{\{U \geq \frac{1}{n}\}} \in \mathcal{L}^2,$$
$$G_2(\vartheta^n) = \frac{1}{U}(S+U)V^+ I_{\{U \geq \frac{1}{n}\}} \in \mathcal{L}^2,$$

and so $\vartheta^n \in \Theta$ for all n. Moreover, it is evident that $G_2(\vartheta^n)$ converges in $\mathcal{L}^2$ to

$$H = \frac{1}{U}(S+U)V^+ \in \mathcal{L}^2.$$

But the only predictable process ξ with $G_2(\xi) = H$ is $\xi = \frac{1}{U}$ (consider the sets $\{V = \pm 1\}$), and since

$$G_1(\xi) = \frac{S}{U} \notin \mathcal{L}^2,$$

ξ is not in Θ, so H is not in $G_2(\Theta)$ and $G_2(\Theta)$ is not closed in $\mathcal{L}^2$. This ends the example.

As an immediate consequence of Theorem 3, we get

Corollary 4. *If $\widehat{K}_T$ is P-a.s. bounded and satisfies (2.5), there exists a unique solution $\xi^{(c)} \in \Theta$ to the problem*

$$\text{(3.1)} \qquad \textit{Minimize } E\left[\left(H - c - \int_0^T \vartheta_s \, dX_s\right)^2\right] \textit{ over all } \vartheta \in \Theta$$

for every pair $(H, c) \in \mathcal{L}^2 \times I\!R$.

This result answers a previously unresolved question from financial mathematics concerning the existence of a variance-minimizing hedging strategy. Except for the case of finite discrete time completely solved in Schweizer [12], earlier work on this problem was all based on very restrictive assumptions; see Duffie/Richardson [4], Schäl [10], Schweizer [11], Hipp [5], Schweizer [13]. For a slightly more general result, we refer to Monat/Stricker [8]. Like Theorem 3, Corollary 4 is almost sharp: the same example as above shows that (3.1) need not have a solution in general.

A second application concerns the problem of *variance-minimization under restricted information*. For any subfiltration $I\!\!G \subseteq I\!\!F$ satisfying the usual conditions, denote by $\Theta(I\!\!G)$ the set of those $\vartheta \in \Theta$ which are $I\!\!G$-predictable. In answer to a question of D. Heath, we then have

Theorem 5. *If $\widehat{K}_T$ is P-a.s. bounded and satisfies (2.5), then $G_T\big(\Theta(I\!\!G)\big)$ is closed in $\mathcal{L}^2$ for any filtration $I\!\!G \subseteq I\!\!F$ satisfying the usual conditions.*

Proof. If we denote by $B^{p,I\!\!G}$ the dual $I\!\!G$-predictable projection of B and define the $I\!\!G$-predictable processes

$$\varrho^{ij} := \frac{d\left(\int \sigma^{ij} \, dB\right)^{p,I\!\!G}}{dB^{p,I\!\!G}} \qquad \text{for } i, j = 1, \ldots, d,$$

then (1.3) can be rewritten as

$$E\left[\int_0^T \vartheta_s^* \varrho_s \vartheta_s \, dB_s^{p,I\!\!G}\right] < \infty.$$

This shows that $\Theta(I\!\!G)$ is closed in $L^2(M)$, and so the assertion follows as in Theorem 3.

q.e.d.

Corollary 6. *If $\widehat{K}_T$ is P-a.s. bounded and satisfies (2.5), there exists a unique solution to*

$$\text{Minimize } E\left[\left(H - c - \int_0^T \vartheta_s \, dX_s\right)^2\right] \text{ over all } \vartheta \in \Theta(\mathbb{G})$$

for every pair $(H, c) \in \mathcal{L}^2 \times \mathbb{R}$ and every filtration $\mathbb{G} \subseteq \mathbb{F}$ satisfying the usual conditions.

We conclude this paper with some examples where the assumptions of Theorem 3 are satisfied. Of course, (2.5) is trivially fulfilled whenever $\widehat{K}$ is continuous. This is always the case if X is continuous, so that it then only remains to check boundedness of $\widehat{K}$. (Actually, this is always sufficient; see Monat/Stricker [8,9].) Another example is provided by the multidimensional jump-diffusion model considered in Shirakawa [15,16], Xue [17] and Schweizer [13], among others. As a special case, this includes the multidimensional diffusion model introduced by Bensoussan [1] and studied for instance by Karatzas/Lehoczky/Shreve/Xue [7]. In the one-dimensional case, this model reduces to

$$dX_t = \mu_t X_t \, dt + \sigma_t X_t \, dW_t,$$

and since

$$\widehat{K}_t = \int_0^t \frac{\mu_s^2}{\sigma_s^2} \, ds,$$

Corollary 4 gives an existence result for (μ_t/σ_t) *bounded*. This is a clear improvement over previous results which typically required this ratio to be *deterministic*; see Schweizer [11].

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