How does one write a mathematical text or paper?

For students, this question typically comes up in connection with either a written presentation for a seminar, or a Bachelor thesis, or a Master thesis. The basic requirements are the same in each of these cases: The overall goal in such a context is to work out and present, under some guidance, a mathematical problem and its solution. There are a number of general rules for good exposition, including the following:

1) A mathematical text (paper, for short) should be *self-contained*. Hence one usually has to define used terms and notations within the paper. Alternatively, they can be quoted from a standard source if this does not hamper the reader's understanding.

2) A *linearly structured exposition* is almost always best, and this has implications. Using terms before they are defined is bad style and makes difficult reading. The same holds for complicated structures and long sentences. In particular, a reasoning of the form "A implies B" is much clearer and simpler than a formulation like "we have B, where we have used A".

3) A consequence of linear structuring is that it helps to create clear writing because it forces one to think ahead. Suppose that A implies B, that A follows from A1 and A2, and that B implies B1 and B2. Compare the following two formulations:

"A1 together with A2 implies A. This yields B, which in turn leads to B1 as well as B2."

"We have B because this follows from A. To obtain A, we use A1 and A2. Note also that B1 and B2 both are consequences of B."

Which is clearer? And which is easier to write down because one can just add sentences as one goes along? The difference quickly becomes even bigger if the involved statements and connections are more complicated.

4) *Notations* should be as short and yet as informative as possible. Within a paper, notation should be *consistent*, and so one symbol usually cannot denote different objects.

5) A paper must be *mathematically correct*. One can use heuristics or intuitive arguments, but they must be clearly distinguished from exact statements and proofs, and this must be made evident by an appropriate formulation. In mathematical statements, it must be clear what the assumptions are, and what the conclusions.

6) All results should have clearly stated assumptions and conclusions, and the proofs should make it visible where the assumptions are used. A theorem without assumptions is incomplete, and one cannot expect a reader to remember all general assumptions that may have appeared earlier.

7) Mathematical *assertions* must either be obvious or be *proved* carefully and rigorously. For the latter, one can either give a complete proof or provide a precise reference to an exactly quoted work in existing literature. In the second case, part of the task is to check whether (and perhaps explain why) the assumptions made in the quoted source are satisfied in the given context, unless this is completely evident.

8) Finding the right amount of details to provide needs a good balance. If B follows from A after writing out several steps, then a formulation like "A implies B" is woefully too short. On the other hand, if $a = b$ and $x = y$, then writing $a^2 - x^2 = (a-x)(a+x) = (b-x)(b+x) = (b-y)(b+y) = b^2 - y^2$ is obviously too complicated.

9) A paper should be *formulated in proper language* and in particular consist of complete

and grammatically correct sentences. A collection of formulas interpolated by interspersed comments and phrases is not acceptable. A well-written paper is a pleasure to read.

10) A paper should be *clearly structured*. It typically starts with an introduction, contains a number of sections or chapters, and ends with a list of references. Especially in the case of a longer paper, it is very helpful if each chapter begins with a short explanation of what is done in that chapter and how it fits into the overall framework. If one has a lot of notations, it may be useful to summarise the important ones compactly in one place. Alternatively, one can recall notations that have been introduced earlier but have not been used for a while.

11) The *introduction* is extremely important and (especially for a Bachelor or Master thesis) should not be underrated. Its formulation must be precise, but still understandable for non-experts, and readers should be able to follow the introduction without already knowing in detail what the paper is about. The goal of the introduction is to provide an overview of the paper or text to an interested reader, and it should help him decide whether he wants to continue reading or not. This requires a clear and correct exposition without using unnecessary technical details or notations. At the same time, the introduction should point out the essential mathematical concepts, ideas and arguments contained in the paper. A badly written introduction is likely to kill the reader's interest at a very early stage.

12) Comparisons to other literature are best done later rather than earlier. If all the notations and results have already been presented, it is much easier to make (and understand) comparisons than at the beginning where many concepts have not yet been introduced.

13) If you are not really convinced of the quality of your exposition, it is better and more efficient to reread and rework your text one more time, instead of handing in an unfinished version. This will save time, nerves and effort for both reader and author.

One final comment: Writing a good paper takes a lot of time and effort. But it pays, because a well-written paper is more widely read. And in the end, it also saves time — every reader is able to read a good paper much more quickly than a bad one. As we are all readers much more often than we are authors, the potential savings in time are enormous.

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