Scientific works in mathematics

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ETH Zürich – DMATH

December 11th, 2019

Important! Do not leave until your student card has been checked to confirm attendance!

Lunch Sessions - Thesis Basics für Mathematik-Studierende

Aufbau der Lunch sessions				
Basic 1	Montag, 9.3.2020	**Starterkit – Tipps für den Einstieg in die schriftliche Arbeit**	**Sigrid Freudl Leitung MathBib**	30‘
Basic 2	Dienstag, 10.3.2020	**Mathematische Datenbanken zbMATH und MathSciNet**	**Flavia Lanini Fachreferentin Mathematik ETH-Bibliothek**	30‘
Basic 3	Mittwoch, 11.3.2020	**LaTeX-Basics**	**Peter Kessler Informatikdienste ETH**	30‘
Basic 4	Freitag, 13.3.2020	**LaTeX-Workshop**	**Peter Kessler**	60-90‘

Outline

- Types of mathematical works
- Publication standards in pure and applied mathematics
- Data handling
- Ethical issues
- Citation guidelines
- References

Warning!

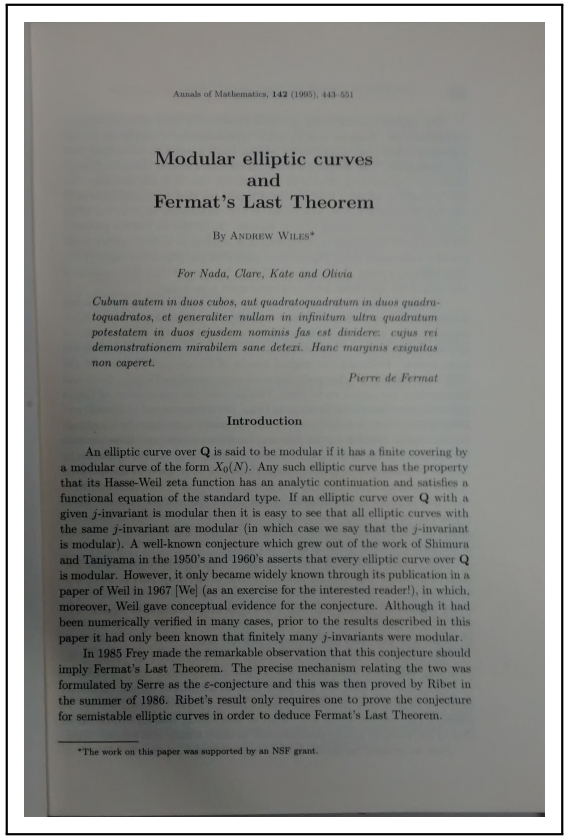
- This presentation is not exhaustive.
- It applies really only to mathematics.
- It is not meant to be definitive, and only presents some guidelines.
- Always discuss with a mentor if in doubt.

Types of mathematical works

Mathematics is made public as:

- Research papers (in journals or conference proceedings);
- Research monographs or textbooks;
- PhD theses;
- Survey-type papers, including (most) master or bachelor theses;
- Software.

A *research paper* is a self-contained article presenting one or more **new** results in mathematics; these results must be proved completely according to the standards of mathematical rigor.



Annals of Mathematics, **142** (1995), 443–551

Modular elliptic curves and Fermat's Last Theorem

By Andrew Wiles*

For Nada, Clare, Kate and Olivia

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos ejusdem nominis fas est dividere: cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

Pierre de Fermat

Introduction

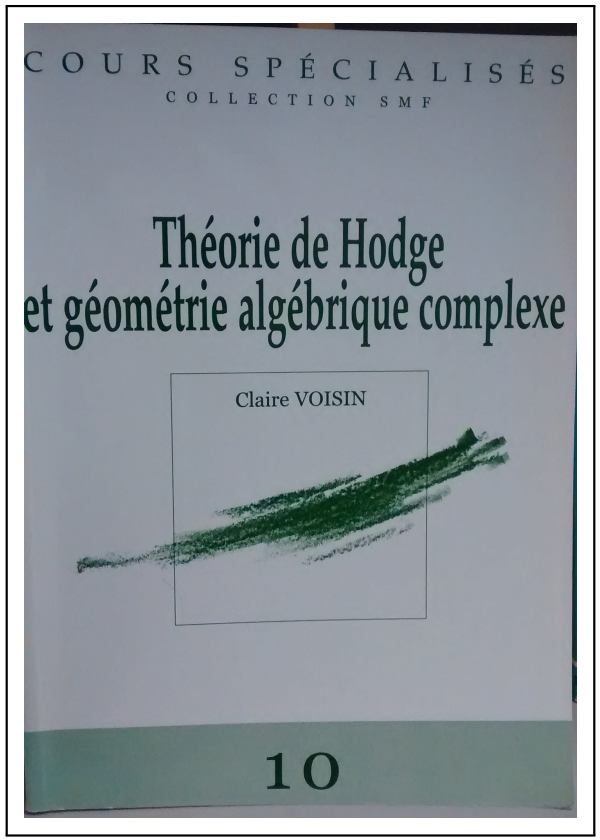
An elliptic curve over $\mathbf{Q}$ is said to be modular if it has a finite covering by a modular curve of the form $X_0(N)$. Any such elliptic curve has the property that its Hasse-Weil zeta function has an analytic continuation and satisfies a functional equation of the standard type. If an elliptic curve over $\mathbf{Q}$ with a given j-invariant is modular then it is easy to see that all elliptic curves with the same j-invariant are modular (in which case we say that the j-invariant is modular). A well-known conjecture which grew out of the work of Shimura and Taniyama in the 1950's and 1960's asserts that every elliptic curve over $\mathbf{Q}$ is modular. However, it only became widely known through its publication in a paper of Weil in 1967 [We] (as an exercise for the interested reader!), in which, moreover, Weil gave conceptual evidence for the conjecture. Although it had been numerically verified in many cases, prior to the results described in this paper it had only been known that finitely many j-invariants were modular.

In 1985 Frey made the remarkable observation that this conjecture should imply Fermat's Last Theorem. The precise mechanism relating the two was formulated by Serre as the ε-conjecture and this was then proved by Ribet in the summer of 1986. Ribet's result only requires one to prove the conjecture for semistable elliptic curves in order to deduce Fermat's Last Theorem.

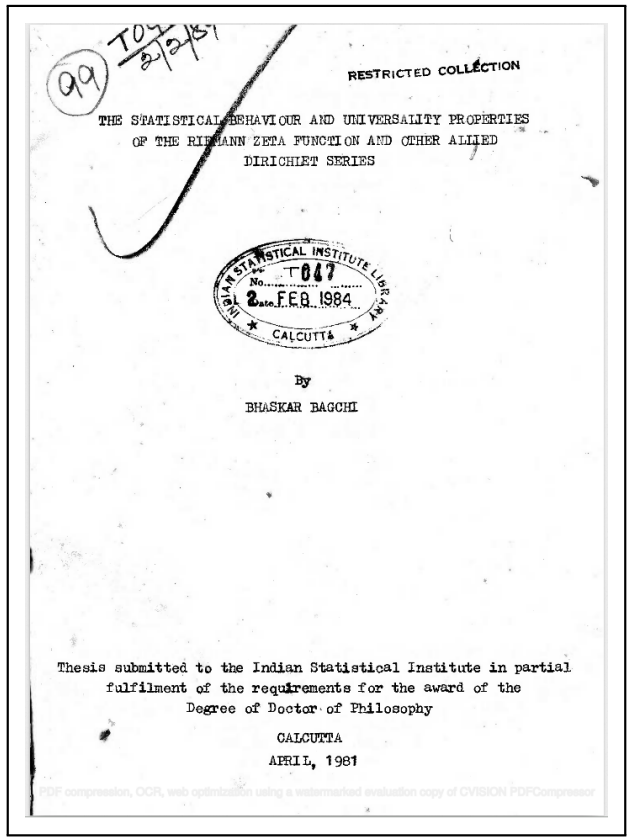
*The work on this paper was supported by an NSF grant.

In statistics, a paper can also deal with new software or applications of statistics.

A *research monograph* is a book on a mathematical topic; it may contain new original research, but it can also be a presentation of known results and methods.



A *PhD thesis* in mathematics contains one or more **new** results with complete proofs, often presented with more details and background than in a research article, and sometimes combined with surveys of known material.



RESTRICTED COLLECTION

THE STATISTICAL BEHAVIOUR AND UNIVERSALITY PROPERTIES OF THE RIEMANN ZETA FUNCTION AND OTHER ALLIED DIRICHLET SERIES

By

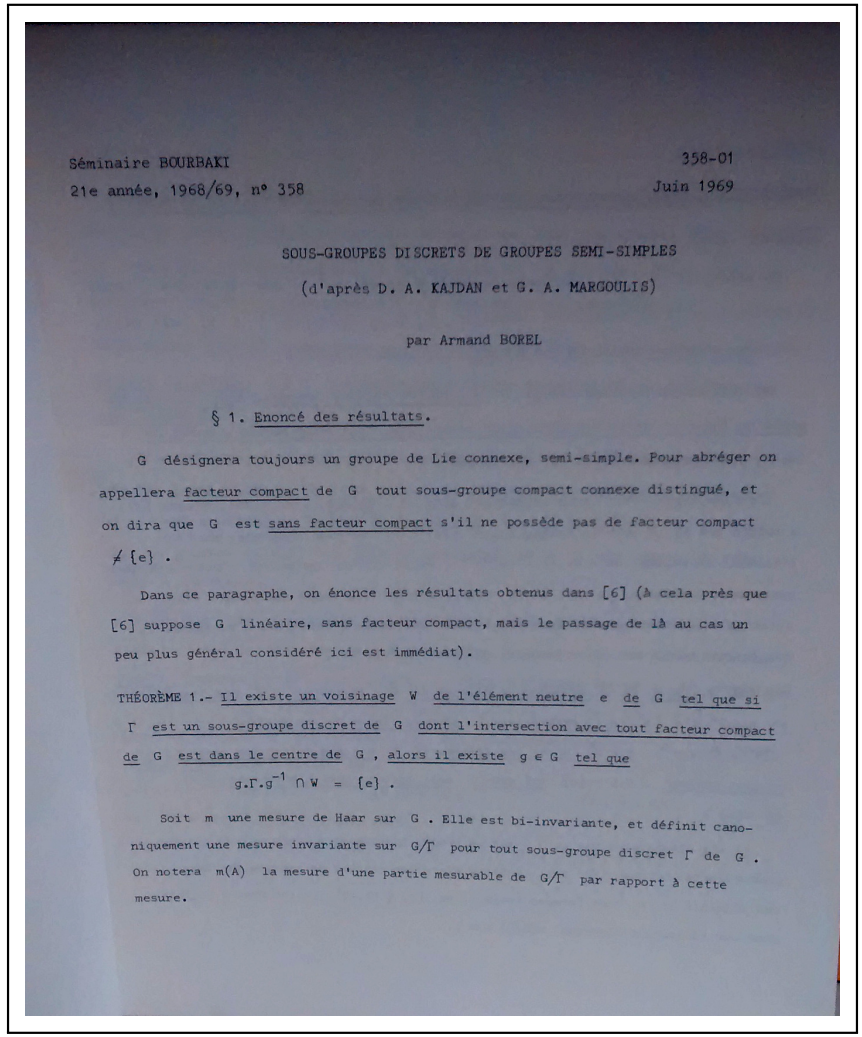
BHASKAR BAGCHI

Thesis submitted to the Indian Statistical Institute in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy

CALCUTTA

APRIL, 1981

A *survey* is an article that presents mostly known results, either with proofs or in an informal way.



Séminaire BOURBAKI 358-01
21e année, 1968/69, nº 358 Juin 1969

SOUS-GROUPES DISCRETS DE GROUPES SEMI-SIMPLES

(d'après D. A. KAJDAN et G. A. MARGOULIS)

par Armand BOREL

§ 1. Enoncé des résultats.

G désignera toujours un groupe de Lie connexe, semi-simple. Pour abréger on appellera facteur compact de G tout sous-groupe compact connexe distingué, et on dira que G est sans facteur compact s'il ne possède pas de facteur compact $\neq \{e\}$.

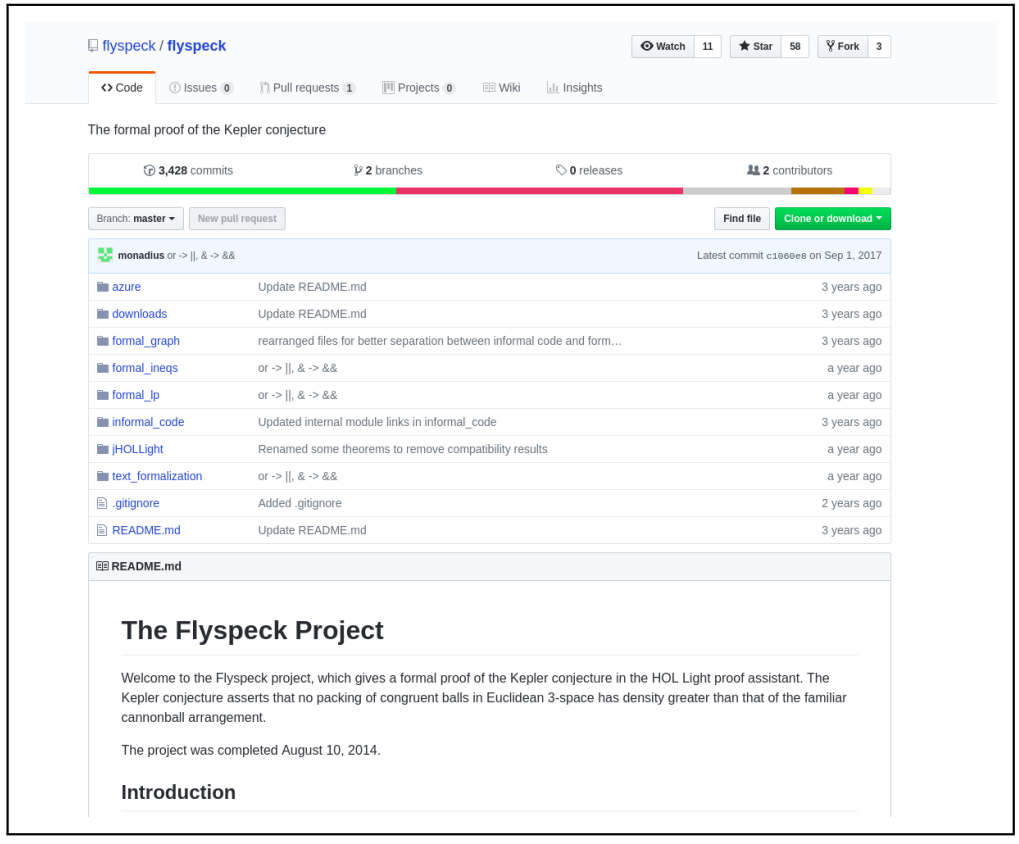
Dans ce paragraphe, on énonce les résultats obtenus dans [6] (à cela près que [6] suppose G linéaire, sans facteur compact, mais le passage de là au cas un peu plus général considéré ici est immédiat).

THÉORÈME 1.- Il existe un voisinage W de l'élément neutre e de G tel que si Γ est un sous-groupe discret de G dont l'intersection avec tout facteur compact de G est dans le centre de G , alors il existe $g \in G$ tel que

$$g.\Gamma.g^{-1} \cap W = \{e\} .$$

Soit m une mesure de Haar sur G . Elle est bi-invariante, et définit canoniquement une mesure invariante sur G/Γ pour tout sous-groupe discret Γ de G . On notera m(A) la mesure d'une partie mesurable de G/Γ par rapport à cette mesure.

- Software may be a computer program used to prove a result contained in a research paper (“computer-assisted proofs”);
- It may be a computer program implementing a new algorithm or giving a new implementation of a known algorithm.
- In recent years, it may be a *formal proof* of a theorem.



Publication process

- Research papers are often first made available as *preprints* on web sites such as `http://arxiv.org`; the preprint should be complete and fully checked by the author(s).

- Research papers are then usually submitted for publication, either to a specialist mathematical journal, or as a chapter of a proceedings volume for a conference.
- In principle, the results of a research paper are considered to have been checked and verified for correctness only after it is accepted for publication, after full refereeing by one or more experts.

Publication standards: "pure" mathematics

- The most prestigious papers appear in generalist mathematical journals, or in journals specializing in some specific area of mathematics (e.g., analysis, combinatorics, number theory, etc); proceedings of conferences are, usually, not as important;
- If there is more than one author, they are listed *in alphabetical order*, with no implied ordering concerning the share of the work done by the various authors.
- The thesis advisor (for a PhD thesis) or mentor (for a postdoctoral researcher) or head of group or institute does not usually appear as an author, unless he or she has contributed scientifically at the same level as other authors.

Special features: statistics

- Prestigious papers on theory appear in mathematical journals focusing on statistics; prestigious papers involving a new method **and** applications appear also in journals such as *Nature Methods*;
- If there is more than one author, the listing of the names usually has some meaning. In particular, the first author contributed most and the last author is most senior and usually gave most strategic input (he or she is not listed if he or she did not contribute to the paper). The authors mentioned between first and last contributed less.
- Simulation studies must be archived in a way that a third person can reproduce them.

Special features: operations research

- Prestigious papers may also appear in a highly competitive proceedings volume;
- Scientific computations must be reproducible, best practice being to make the computer code publicly available.

Numerical experiments in applied mathematics

- Numerical experiments **must** be fully reproducible: any reader of the paper should be able to reproduce all the results from the paper the code and data available online.
- The computer code is part of the scientific work, hence it should appear in an Appendix (if short enough) or in a repository that is publicly available and has a guarantee of long-term availability. The code must come with full documentation and with all instructions on how to install and run it.

Citing software

- Third party software or code libraries used for scientific work should be mentioned;
- The URL of the software's website should be given in a footnote or a reference of the form

  [1] BETL - Boundary Element Template Library, URL: `http://www.sam.math.ethz.ch/betl/`, accessed March 2015

  It is important to mention an access date, because websites may not be persistent.
- The website of the software often suggests how to cite it, e.g., by asking to cite a specific peer-reviewed paper. These suggestions should be followed and the paper should be cited, in addition to providing the URL.
- For really standard software such as compilers (GCC, CLANG), or MATLAB, Octave, Mathematica, MAPLE, etc, it may be enough to mention the name, since these names are "brands".

Ethical issues

- One should not claim or announce a result without having a complete proof and having checked it (as far as possible, since mistakes are always possible);
- Priority for proving a result is not directly linked to publication, and may be established by making available a (fully detailed) preprint, or by having a thesis manuscript;
- All authors of a research paper must have made a *significant scientific contribution* to the new results that it contains;
- All results that are used or other information or insights that have been involved in the research represented by the paper must be properly acknowledged.

Data handling in general

- More and more data is created by scientists, even in mathematics.
- Data handling needs to be carefully specified.
- Funding agencies (such as the Swiss SNF) today often require a precise "Data Management Plan", according to the "FAIR" principle (Findable, Accessible, Interoperable and Reusable).

Citation guidelines

- The following guidelines apply (at least) to pure mathematics and statistics. For more details and examples, see the MathBIB Moodle module.
- Citations must have a sound scientific purpose, and in particular an author should not cite his or her own work, or that of friends or colleagues, without good reason.
- Any citation of a specific, precise, result, must be accompanied with a precise location in the paper or book that is referenced.

Standard result

The citation need not belong to the original paper where the result is proved, but to a later account. It is then usually clear to the reader that the authors of the work referenced are not the discoverers of the theorem.

Example. To cite the Banach-Steinhaus Theorem, supposing that one wishes to use Bourbaki's "Elements of Mathematics" as the reference, one should write:

> By the Banach-Steinhaus Theorem ([1, EVT, III, §4, Cor. 2]), we have...

and not

> By the Banach-Steinhaus Theorem [1], we have...

Background information

If a book or paper is cited only to provide background information, it may be cited without more precision.

Example.

Signs of Fourier coefficients of cusp forms have also been studied by Matomäki [Mat] and Ghosh-Sarnak [G-S].

For a general introduction to Hodge theory, see for instance the book of Voisin [V].

Spelling out names

When citing for the first time, it is often best to spell out the names of the author(s) explicitly.

Example.

> It was proved by Fouvry [13] and Bombieri, Friedlander and Iwaniec [5] that certain arithmetic functions have exponent of distribution strictly larger than $1/2$...

instead of

> It was proved in [5,13] that certain arithmetic functions have exponent of distribution strictly larger than $1/2$...

Attribution

A theorem which is stated without specific attribution is *usually supposed to have been proved by the author of the text*. If this is not the case, precise attribution is needed.

Example.
Write

> We will prove in this text:
> **Theorem** (Dirichlet). *Let a be an integer and let $q \geq 1$ be an integer such that a and q are coprime. Then there are infinitely many primes p congruent to a modulo q.*

and not

> We will prove in this text:
> **Theorem.** *Let a be an integer and let $q \geq 1$ be an integer such that a and q are coprime. Then there are infinitely many primes p congruent to a modulo q.*

Adapting previous arguments

It is acceptable in a *research paper* to follow closely the proof of an already published work to prove *an analogue result*, but this fact must be clearly indicated with proper reference.

Example.

> This lemma is a slight variant of a result of Helfgott [H2004, Lemma 5.1], and our proof follows his argument closely.

Acknowledging input from other works

A mathematical text should also cite and acknowledge works or ideas which have had an important influence in the search for the proof.

Example.

> The basic idea for this was communicated to me by a referee for *Inventiones* in connection with the publication of my paper [6], and I then had only to translate it from Hodge theory into étale cohomology. I would therefore like to heartily thank this referee...

(actual quote from G. Faltings) or

> The author learned about the technique in this proof from discussions with Venkatesh concerning sparse equidistribution problems.

References

- These slides

  people.math.ethz.ch/~oezlemi/pdf/scientific-works-slides-11-12-19.pdf

- Course of Prof. W. Merry in HS 2019, *"Communication in mathematics"*;

  https://www.merry.io/courses/communication-in-mathematics/

  information on the course catalogue:

  vvz.ethz.ch

- MathBIB Moodle:

  moodle-app2.let.ethz.ch/course/view.php?id=519

- "Richtlinien für Integrität in der Forschung und gute wissenschaftliche Praxis an der ETH Zürich", Rechtssammlung, RS 414 (German and English translation):

  rechtssammlung.ethz.ch

- SNF guidelines for data handling:

  www.snf.ch/en/theSNSF/research-policies/open_research_data/