Finite group schemes

Lecture course in WS 2004/05

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Outline

The aim of the lecture course is the classification of finite commutative group schemes over a perfect field of characteristic p, using the classical approach by contravariant Dieudonné theory. The theory is developed from scratch; emphasis is placed on complete proofs. No prerequisites other than a good knowledge of algebra and the basic properties of categories and schemes are required. The original plan included p-divisible groups, but there was no time for this.

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