LATTICES IN LIE AND LOCALLY COMPACT GROUPS

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I propose to give an advanced graduate course about the theory of lattices in locally compact groups including both the classical theory that has been developed mainly during the 60's and the 70's of the previous century as well as recent and present progress.

1. Table of content

(1) Basic properties and examples (geometric and arithmetic).
(2) Malcev theory for lattices in Nilpotent groups.
(3) Lattices in Solvable groups: Mostow's theory and recent progress.
(4) The classical theory of lattices in semisimple groups.
(5) Rigidity and Arithmeticity.
(6) Bader–Furman theory: Generalized Weyl groups and new approach for rigidity.
(7) The topology of locally symmetric manifolds.
(8) Invariant random subgroups and local convergence of locally symmetric spaces.

2. description

The most familiar examples of lattices are arithmetic groups and fundamental groups of manifolds. I'll start by discussing these example and proving some basic properties. Then I'll proceed to describe the elementary, yet beautiful, theory of lattices in Nilpotent groups. Lattices in solvable groups are much harder to deal with. I'll show an elementary proof for Mostow's theorem about uniformity of lattices in Lie groups and discuss some extensions of this result to more general locally compact solvable groups [5]. Surprisingly it turns out that in general locally compact solvable groups may admit non-uniform lattices. Then I'll move on to describe the classical theory of lattices in semisimple groups including: Borel density theorem, Kazhdan–Margulis theorem, property (T), finite presentability, variety of deformations, local rigidity, Wang's finiteness theorem etc. One of my aims is to describe new proofs for some of these classical results. Then I'll discuss Mostow's and Margulis' rigidity theorems and some recent generalizations, again adopting a more modern approach following the work of Bader and Furman. I may also dedicate some time to describe the beautiful work of Caprace and Monod about lattices in CAT(0) spaces. The last part of the course will be devoted to the study of the topology of locally symmetric manifolds $\Gamma\backslash G/K$. I'll speak about Gromov's linear bound on the Betti numbers, as well as a stronger recent result that gives a uniform Lück approximation of higher rank symmetric spaces [1] — somewhat surprisingly, this strong result is proved by adopting an approach introduced

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by Benjamini and Schramm for probabilistic, or local, convergence for sequences of finite graphs, or in our case, Riemannian manifolds.

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