POSTMODERN ALGEBRA: OPERADIC CALCULUS

JEAN-LOUIS LODAY

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Résumé

In 1930 appeared the first edition of the influential book "Moderne Algebra" by B.L. van der Waerden, a former Zürich Professor. The topic was essentially commutative-associative algebras. Since then many other types of algebras have cropped up in different parts of mathematics. Besides the "three graces": commutative, associative and Lie, we can name : Poisson, Jordan, Hopf, A-infinity, Batalin-Vilkovisky, L-infinity, Leibniz, dendriform, and so on. The relevant tool to study all these types of algebras and their relationship is called an *operad*. The idea is to work, not with the elements of a given algebra, but with all the operations that one can perform on these elements. So the product of elements is replaced by the composition of operations. Translated in terms of generating series it means that we are interested into the composition of series rather than into the product of series.

This subject, which was unearthed by algebraic topologists in the 70's, found a renaissance in the 90's under the influence of Maxim Kontsevich.

The aim of the course is to introduce the notion of operads and to explain one of the main tools of the theory which is Koszul duality. This topic bears relationships with differential geometry, noncommutative geometry, algebraic topology, combinatorial algebra, deformation theory, renormalization theory, rewriting systems. Some of these links will be treated, depending on the wishes of the audience.

References

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Institut de Recherche Mathématique Avancée, CNRS et Université de Strasbourg, 7 rue R. Descartes, 67084 Strasbourg Cedex, France

E-mail address: loday@math.u-strasbg.fr